PSERC

Electric Power Transfer Capability: Concepts, Applications, Sensitivity, Uncertainty

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ELECTRIC POWER TRANSFER CAPABILITY: CONCEPTS, APPLICATIONS, SENSITIVITY AND UNCERTAINTY

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Chapter 1

Introduction

1.1 Summary

Transfer of bulk electrical power over long distances is routine in North America in order to have a reliable and economical electrical supply. For example, hydro-electric power generated in Canada can be transferred to consumers and industry in Los Angeles using the high voltage transmission system. But the transmission system has a limited capability to transfer power. The maximum power that can be transferred is called the transfer capability. To operate the power system safely and to gain the benefits of the bulk power transfers, the transfer capabilities must be calculated and the power system planned and operated so that the power transfers do not exceed the transfer capability.

The purpose of this document is to explain concepts and calculations of transfer capability and describe applications of transfer capability. The document aims to give a tutorial introduction to some standard transfer capability concepts and introduce some new methods in transfer capability sensitivity and accounting for uncertainty.

Some highlights of the document are:

1. Explanation and illustration of transfer capability using a transfer capability calculator available on the web (chapter 1).

2. Discussion of transfer capability computations and applications (chapters 2 and 4).

3. Fast methods to compute transfer capability sensitivities to a wide range of parameters using formulas; testing of these methods on a 3357 bus system (chapter 3).

4. Extension of standard DC load flow transfer capability methods to AC load flow models and parameters (chapter 3).

5. New method of quantifying uncertainty in transfer capability computations (chapters 5 and 6).

6. Accounting for uncertainty in key parameters (e.g., forecast temperature) in transfer capability computations (chapter 6).

7. Maximizing transfers subject to a given risk of flowgate congestion due to uncertainty (chapter 6).

8. Estimate probability of transmission congestion in any flowgate (chapter 6).

1.2 General motivation

Long distance bulk power transfers are essential for an economic and secure supply of electric power in North America.

Power system transfer capability indicates how much interarea power transfers can be increased without compromising system security. Accurate identification of this capability provides vital information for both planning and operation of the bulk power market. (Planners need to know the system bottlenecks and system operators must not implement transfers which exceed the calculated transfer capability.) Repeated estimates of transfer capabilities are needed to ensure that the combined effects of power transfers do not cause an undue risk of system overloads, equipment damage, or blackouts. However, an overly conservative estimate of transfer capability unnecessarily limits the power transfers and is a costly and inefficient use of the network. Power transfers are increasing both in amount and in variety as deregulation proceeds. Indeed, such power transfers are necessary for a competitive market for electric power. There is a very strong economic incentive to improve the accuracy and effectiveness of transfer capability computations for use by system operators, planners and power marketers.

The practical computations of transfer capability are evolving. The computations presently being implemented are usually oversimplified and in many cases do not take sufficient account of effects such as interactions between power transfers, loop flows, nonlinearities, operating policies, and voltage collapse blackouts. The goal of the methods described here is to improve the accuracy and realism of transfer capability computations. The power system must be operated with some conservatism to account for the effects of uncertainty in power system data. This uncertainty can be analyzed and quantified to provide a defensible basis for the conservatism.

The limitations on power system performance that we consider in this document are transmission line flow limits, voltage magnitudes and voltage collapse. All these limits can be handled in an AC load flow power system model. We do not address limits due to transient stability or oscillations; in our framework, these limits have to be crudely approximated by flow limits.

1.3 A simplified transfer capability calculation

One way to explain transfer capability is to work top-down from the purpose of transfer capability by discussing definitions and meanings of various components of transfer capability and then explaining the various methods for calculating each component. This useful approach is followed, for example, by NERC in defining transfer capability [89], [90]. In this tutorial, we take an alternative approach of explaining how to calculate transfer capability in a simplified case and then generalizing the calculation. We think that a focus on what one typically calculates tends to reveal the meanings and limitations of transfer capability concepts.

This section introduces and explains transfer capability in a simplified context. The transmission network is assumed to be fixed so that, if any contingencies such as a line outage are considered, they are incorporated into the network and do not change during the discussion. Only one transfer capability calculation is explained; in practice the calculation is repeated with a range of assumptions. The modeling is assumed to be deterministic; probabilistic effects are neglected. A more detailed explanation of transfer capability is in Chapter 2.

There are many assumptions or choices made during a transfer capability calculation which can greatly influence the answer. In addition to choices made in modeling the power system, the base case, the transfer itself, and the limiting case must all be chosen. These choices are now described.

Base Case: The base case is an assumed power system operating condition to which the transfer is applied. That is, the base case is the assumed power system operating condition when no power has been transferred. A power system operating condition is generally obtained by specifying the powers generated or consumed at each bus and the control settings and then solving power system equations to calculate the other power system quantities such as power flows on the transmission lines. The base case is assumed to be an operating condition solved in this way and also a secure condition so that all quantities such as line flows and bus voltage magnitudes lie within their operating limits.

Specifying the transfer: A transfer is specified by changes in power injections at buses in the network. For example, a point to point transfer from bus A to bus B is specified by increasing power at bus A and reducing power at bus B. In particular, if 100 MW are to be transferred from A to B, then power at bus B is reduced by 100 MW and power at bus A is increased by 100 MW plus an amount to cover the change in losses. In this case bus A is called a source of power and bus B is called a sink of power.

Limiting Case: A solved transfer limited case is established at which the transfer has been increased to such a value that there is a binding security limit. The binding security limit can be a limit on line flow, voltage magnitude, voltage collapse or other operating constraint. Further transfer in the specified direction would cause a violation of the binding limit and compromise system security.

Summary and transfer margin: Now we can summarize the choices made in the calculation and define the transfer margin:

1. Establish a secure, solved base case.

2. Specify a transfer including source and sink assumptions.

3. Establish a solved transfer-limited case and a binding security limit.

4. The transfer margin is the difference between the transfer at the base case and the limiting case.

One way to compute transfer capability with a software model is called continuation. First, the base case is solved. The amount of the transfer is a scalar parameter which can be varied in the model. (Note that the transfer must be properly defined in terms of bus power injections to implement the amount of transfer as a scalar parameter in the model.) Then the amount of transfer is gradually increased from the base case until a binding limit is encountered. This continuation process requires a series of power system solutions to be solved and tested for limits. The transfer capability is then the change in the amount of transfer from the base case transfer.

Continuation can be simply done as a series of load flow calculations for increasing amounts of transfers or by more elaborate high performance numerical methods. Some continuation software can accurately take account of power system nonlinearity, operator actions, controls such as tap changes, and generator limits as the transfer is increased. On the other hand, useful transfer capabilities can also be obtained with simpler power system models such as the DC load flow approximation.

1.4 AC load flow example using calculator

An interactive transfer capability calculator is available on the web at

http://www.pserc.cornell.edu/tcc/

The calculator uses a continuation method to compute transfer capability. The power system model is an AC power flow model; it represents real and reactive power flows and power system nonlinearity. Operational limits relating to transmission line flow, voltage magnitude, and voltage collapse are represented.

This section assumes that the reader is viewing the calculator on the web.

1.4.1 Getting started on the calculator

When the calculator is first viewed, a base case for a particular 6 bus power system and a transfer from generator bus 1 to load bus 5 is assumed. The transfer capability for this case is calculated simply by clicking the button marked CALCULATE. The transfer capability of 133.23 MW is calculated and the results show that the limiting condition is a flow limit on the line from bus 1 to bus 5. The transfer capability result is also shown graphically as a horizontal bar of length 133.23 MW.

The calculator allows the power system, the base case, and the transfer to be changed. Many of the transfer capability terms used in the calculator are explained by links on the calculator page.

1.4.2 Quickly computing changes to transfer capability

It is useful to know the transfer capability when changes are made in the base case parameters. The changes in the base case parameters of interest can represent variation in simultaneous transfers, assumed data, and system controls.

One way to investigate this is simply to change the base case parameter and rerun the calculator to get the new transfer capability. The disadvantage of this approach is the considerable computation required every time the AC continuation load flow is executed. If, as common in power systems, there are many parameter changes to be considered, then this approach becomes computationally burdensome or impractical. (On the small, 6 bus system, the calculator is quite quick and this problem is less apparent. Running the calculator on the 3357 bus system gives a better appreciation of this point.)

Quick estimates of the transfer capability when parameters are changed can be evaluated by specifying the desired parameter change with the three pop-up menus as indicated. The calculator then displays the estimated transfer capability in text and graphics. The parameter that was changed appears as the vertical axis of the graph.

The estimates are quick because they are obtained from formulas for the transfer capability sensitivity (the transfer capability is NOT calculated again from scratch with the parameter changed). These formulas essentially specify the slope of the gray line in the plot and then the gray line is used to estimate the transfer capability. This procedure is a linear approximation which is accurate for very small parameter changes and less accurate for large parameter changes.

The quick estimates are derived from transfer capability sensitivity formulas. Chapter 3 shows how to compute these sensitivities very quickly for a wide range of parameters. The calculator illustrates the use of these sensitivities to estimate the effect on the transfer capability of changes in parameters.

To find out how accurate these quick estimates are for the selected parameter change, use the button to "verify the estimated transfer capability". The calculator will then rerun the transfer capability from scratch with the selected parameter change to obtain an "actual" transfer capability which can then be compared to the "estimated" transfer capability.

The following subsections explain some of the concepts that can be demonstrated with the calculator.

1.4.3 Transfer capability depends on assumptions

This apparent but important fact can be easily demonstrated and quantified with the calculator by varying the base case. Redispatch of generation or outage of a transmission line can have a particularly marked effect on the transfer capability.

1.4.4 Interactions between transfers

The transfer capability calculation assumes a base case in which other power transfers are happening. It is of interest how the value of these other transfers affects the calculated transfer capability. In particular, the calculator offers a choice of redispatches from a list that are equivalent to transfers and it is of interest to vary these redispatches to determine their impact on the selected transfer capability.

Consider the lossless case for simplicity. Then a transfer of 10 MW from bus 1 to bus 2 increases generation at bus 1 by 10 MW and decreases generation at bus 2 by 10 MW whereas a redispatch of 10 MW from bus 1 to bus 2 decreases generation at bus 1 by 10 MW and increases generation at bus 2 by 10 MW. Redispatch of power has opposite sign to a transfer of power: a redispatch of power from bus 1 to bus 2 is equivalent to a transfer of power from bus 2 to bus 1.

Changes in transfer capability due to redispatch may also be quickly estimated from a solved limiting case using the calculator.

1.4.5 6 bus system

The 6-bus system is a simple power system network loosely based on tutorial cases from the NERC Web site and the text [94]. The 6-bus system is intended to illustrate in a simple context notions of transfer capability and the impact that various actions have on the given transfer capability.

Buses 1, 2, 3 are generators and buses 4, 5, 6 are loads. The general flow of power is, of course, from the generators to the loads but there is also a tendency for power to flow mainly from the lower numbered buses to higher numbered buses. Reactive demand by the loads is large. The network has 11 branches and each branch represents a transmission line.

Now we consider the transfer from generator bus 2 to generator bus 3, (that is, generator 2 increases its supply and generator 3 decreases its supply). The calculator is run with this transfer selected and all other parameters at their base case values. The transfer capability is 208.7 MW and the limiting event is the flow limit joining bus 2 to bus 3.

Now we study the impact of changing parameters on the transfer capability of 208.7 MW.

- The redispatch between any other two generators can have an impact on a transfer capability. Sometimes this effect is small. For example, changes between generator 1 and generator 2 have a small but noticeable effect on the transfer between 2 and 3 (about 12%). Here this 12% means that for every 1.0 MW of redispatch between 1 and 2 there is an increase of 0.12 in the transfer capability between 2 and 3. In other cases, as in the case of redispatch from 1 to 3, the effect on the transfer capability can be large (about -87%). The effect can be either to increase or to decrease the transfer capability.

- Of course, a redispatch from generator 2 and to generator 3 has a 100% impact on the transfer capability between the same two generators. For example, redispatching 10 MW from generator 2 to generator 3 means that generator 2 power output is reduced by 10 MW and generator 3 power output is increased by 10 MW. The effect of this redispatch of the base case is to increase the transfer capability from 2 to 3 by 10 MW.

- Demand increases of active power demands can have a small impact on the transfers (for example, increases of demand at bus 4 have 9% impact on the transfer capability). They can also have a medium impact (for example, increases in demand at bus 2 have a 42% impact on the transfer capability).

They can also have a large impact (for example, increases in active power demand at bus 6 have a 66% on the transfer capability and a 148% impact on the next limit encountered after the transfer capability binding limit).

- As seen in the previous bullet, the impact, when measured in terms of active power can easily be over 100%. This means that for every 1.0 MW of consumption in bus 6, there is a reduction in the transfer capability from bus 2 to bus 3 greater than 1.0 MW.
- Voltage changes can have a large impact or a small impact on the transfer capability. Interestingly, voltage adjustments can significantly affect the flow limits in some cases, not just voltage limits.
- In this example the impact of reactive power changes on the transfer limits between generators is relatively small, amounting to less than 1 MW impact on the transfer capability for every 10 MVAR of change. As we will see later, however, this is not always the case.

These observations are made based on the quick estimates of changes transfer capability. These estimates, which are based on sensitivity formulas, can lose accuracy as the size of the parameter change increases. The estimates can be checked by rerunning the calculator to verify the estimates. The following observations about the accuracy of the estimates can be made:

- The predictions are quite accurate in all cases provided, although they are more accurate in some cases than in others.
- The predictions are better in cases that do not involve voltage changes.
- The worst predictions are those for the case of a line outage contingency.

1.4.6 39 bus system

The 39-bus system is a standard system for testing new methods. It represents a greatly reduced model of the power system in New England. It has been used by numerous researchers to study both static and dynamic problems in power systems.

The 39-bus system has 10 generators, 19 loads, 36 transmission lines and 12 transformers. The 39-bus system is organized into three areas. (Area 3 contains two portions of the network which are not directly connected.)

The following are some comments and observations about 39-bus system cases that the reader can verify by running the calculator:

- The limiting factors for transfers can be flow-flow, flow-voltage, voltage-flow or voltage-voltage (although no voltage-voltage case occurs in the base case for this example).
- The transfer capability can vary quite widely depending on the desired transfer, from a minimum of 230.3 for the bus 32 to bus 39 transfer case to a

maximum for the cases illustrated of 961.6 MW for the area 1 to area 2 transfer.

- For cases involving inter-area power transfers the predictions of the simplified formulas are extraordinarily good.
- For the 39 bus example there is already a noticeable difference in performance associated with the approximate predictions: they are much faster than the subsequent exact verification.
- An interesting aspect of area transfers is that if the manner in which redispatch is defined differs from the manner in which area interchange is defined, it is quite possible that the impact of a redispatch on a transfer is different from 1 to 1. The estimates using sensitivities are, however, quite good.

1.4.7 NYISO 3357 bus system

The NYISO system is a 3357 bus model of a portion of the North American eastern interconnect. The model contains a detailed representation of the network operated by the New York independent system operator and an equivalent representation of more distant portions of the network.

The calculator includes the NYISO system to demonstrate that the continuation and sensitivity methods apply to systems of a more practical size.

The NYISO base case is intended to be an illustrative example of a severely stressed power system. The base case is produced by artificially stressing the NYISO system near to its operating limits. The base case is motivated by a scenario identified as problematic in the New York Power Pool summer 1999 operating study and includes two 345 kV lines out of service.

This case is derived from the general data available within FERC 715 filings of the New York ISO for the year 1999. It does not necessarily represent actual operating conditions and should not be relied upon to draw conclusions about the system itself. It is intended only to illustrate the performance and capabilities of the method for estimation of transfer capability and the impact of changes in conditions to changes in transfer capability. In particular, no tap adjustments or shunt reactor adjustments are performed as part of these simulations.

1.4.8 Concluding comments

Transfer capabilities are fundamental to appropriate operation of the system. In the past it has been considered that transfer capabilities calculations using AC load flow models are difficult because they require the use of nonlinear tools such as continuation power flows. The calculator demonstrates the use of AC continuation load flow to accurately compute transfer capabilities. Moreover, the calculator demonstrates that the impact on transfer limits of system controls, data and other transactions can be quickly estimated without rerunning the continuation power flow. There are many applications for these quick estimates or the sensitivities used to generate them in the rest of this document.

The Web site allows users to readily experiment with transfer capability calculations and develop intuition as to the effect of data, controls and other transfers. Hopefully, hard to explain concepts such as the impact of voltages on flow limits, or the impact of reactive power injections on remote voltage limits, or even the reasons why 1 MVAR can be worth several MW of transfer capability under some conditions will become clearer to users of this site.

1.5 DC load flow example

A DC load flow model represents real power flows and injections and voltage angles. The limits considered are line flow limits (sometimes other system limits are approximated as equivalent line flow limits). The DC load flow is a linear model; the line flows are a linear function of the powers injected (the DC load flow equations are obtained by linearizing the power flow equations about an operating point). The DC load flow model is lossless and changes in bus injections must always sum to zero.

Implementing the transfer capability computation of section 1.3 simplifies in some ways when the DC load flow approximation is made. The security of the base case can be easily checked by ensuring that all line flows are within their limits. Since the model is linear, the behavior of the line flows as the transfer increases can be linearly extrapolated to find the limiting line flow and the "continuation" process is much easier. Indeed this calculation can be done in terms of generation shift distribution factors as illustrated below.

Consider the DC load flow approximation of the 6 bus example from the text by Wood and Wollenberg [94]. (This 6 bus example has different parameters than the 6 bus example used in the calculator.) The 6 bus example has 3 generators and any transfer can be specified by the 3 power injections at each of the 3 generators. However, since these 3 power injections must sum to zero (the DC load flow model is lossless), any transfer can be specified by the power injections at the generators at buses 2 and 3. For example, a transfer of 10 MW from bus 2 to bus 3 is specified by (-10,10) and a transfer of 5 MW from bus 1 to bus 2 is specified by (5,0)

The complete transfer capability information can be evaluated from the generation shift factors and the margin remaining on each line. These data are shown in Table 1.1 (c.f. Fig. 11.8 in [94]).

The meaning of the generation shift factor of –0.47 for bus 2 and line 1-2 is that an injection of 1 MW at bus 2 and –1 MW at bus 1 will decrease the flow on line 1-2 by 0.47 MW. Similarly, the generation shift factor of 0.22 for bus 3 and line 2-4 implies that an injection of 1 MW at bus 3 and –1 MW at bus 1 will increase the flow on line 2-4 by 0.22 MW. For simplicity, the margins remaining on each line are all chosen to be 10 MW.

Let us compute a transfer capability using Table 1.1. We specify the first transfer as being from bus 1 to bus 2. If the amount of transfer is T MW, then it is specified by injections $(T, 0)$ at buses 2 and 3. Since in this example, all lines have equal margin remaining, the line that will reach its limit first is the line with the largest shift

line	gen. shift factor bus 2	bus 3	margin remaining (MW)
1–2	–0.47	–0.40	10
1–4	–0.31	–0.29	10
1–5	–0.21	–0.30	10
2–3	–0.05	–0.34	10
2–4	–0.31	–0.22	10
2–5	–0.10	–0.03	10
2–6	–0.06	–0.24	10
3–5	–0.06	–0.29	10
3–6	–0.01	–0.37	10
4–5	–0.00	–0.08	10
5–6	–0.06	–0.13	10

Table 1.1: Generation shift factors and margin remaining for a 6 bus system

factor for bus 2, which is line 2-4. The transfer capability is given by $0.31\,T = 10$, or $T = 32.3$ MW.

For another example, specify a second transfer as being from bus 2 to bus 3. If the amount of transfer is T MW, then it is specified by injections $(-T, T)$ at buses 2 and 3. If T MW are transferred, then the changes in the flow on a given line are T (gen shift factor bus 3 - gen shift factor bus 2). Since in this example, all lines have equal margin remaining, the line that will reach its limit first is the line with the largest difference between the generation shift factor for bus 3 and bus 2, which is line 3-6. The transfer capability is given by $(0.37 - (-0.01))\,T = 0.38\,T = 10$, or $T = 26.3$ MW.

The transfer capabilities computed so far assume that only one of the transfers was implemented. If the two transfers are done at the same time or one after another, then they definitely do interact. For example, if both transfers occur simultaneously and at the same rate, then if T is the amount transferred in *one* of the transfers, then the injections at buses 2 and 3 are $(0, T)$. (The simultaneous transfer is equivalent to a transfer from bus 1 to bus 3.) Then the limiting line is the line that will reach its limit first is the line with the largest shift factor for bus 3, which is line 3-6. The transfer capability is then given by $0.37\,T = 10$, or $T = 27.0$ MW.

For further description of generator shift factors and their relation to power transfer distribution factors see section 2.7.

Chapter 2

Transfer capability

2.1 Purpose of transfer capability computations

Generally speaking, the term "transfer capability" refers to the amount of electric power that can be passed through a transmission network from one place to another. The concept of transfer capability is useful for several reasons.

A system which can accommodate large inter-area transfers is generally more robust and flexible than a system with limited ability to accommodate inter-area transfers. Thus, transfer capability can be used as a rough indicator of relative system security.

Transfer capability is also useful for comparing the relative merits of planned transmission improvements. A transmission expansion that increases transfer capability between two areas of the grid might be more beneficial for increasing both reliability and economic efficiency than an alternate improvement that provides a lesser increase in transfer capability.

Along similar lines, transfer capability can be used as a surrogate for more specific circuit modeling to capture the gross effects of multi-area commerce and provide an indication of the amount of inexpensive power likely to be available to generation deficient or high-cost regions.

Transfer capability computations facilitate energy markets by providing a quantitative basis for assessing transmission reservations.

2.1.1 Transfer capability and power system security

Transfer capability computations play a role in both the planning and operation of the power system with regard to system security.

One benefit of interconnected power systems is the potential for increased reliability. In an interconnected system, the loss of generation in one area can be replaced by generation from other areas. Thus, several systems interconnected can survive contingencies that the individual systems could not. Transfer capability computations are useful for evaluating the ability of the interconnected system to remain secure following generation and transmission outages.

Determining the adequacy of the transmission system in allowing external generation to replace internal generation is a typical application for transfer capability computations.

For this purpose, a model of the network reflecting the anticipated conditions is assumed. Several generators within one area are selected as sinks. The power injected to the network at these locations is systematically reduced or eliminated to reflect the planned or unplanned loss of the units. For each generation outage scenario, several external generators are selected as potential sources. The choice of sources and the participation of each source depends upon the assumption concerning the time frame of the response.

The purpose of the transfer capability computation is to determine the quantity of lost generation that can be replaced by the potential reserves and the limiting constraints in each circumstance. In addition to varying the assumptions regarding the generation sources and sinks to reflect different outages and reserve locations, the computations are often repeated assuming different loading conditions or increasing loads and coincident branch element outages.

The previous example illustrates a typical planning or operational study. A very similar application of transfer capability computations is useful for near real time security analysis.

During the course of any day, events or circumstances are encountered that require operators to determine prevention or mitigation actions, or verify that the existing operating guides will perform effectively. Consider a situation in which large transfer across a system is observed during exceptional weather. Although the circumstances are within what might have been considered in planning or had been previously observed, the operators have specific information concerning the actual system state and must anticipate the future state of the system in the coming hours. The operators would like to know how much additional transfer the system could maintain before security is compromised - the security margin.

Based on the energy schedules or typical patterns, the transfer capability for a feasible transfer can be determined. In this situation, the transfer capability represents a security margin. The distance between the present state and a state violating a security criteria is the amount of the transfer that initiates a security violation.

2.1.2 Transfer capability and market forecasting

Among market participants and regulators there is great interest in anticipating the behavior of the electricity markets. However, the detailed operation of the market and network in just one instance requires an enormous quantity of data and this data is not all readily accessible. Given that market participants often want to simulate the market over many thousands of instances introducing variation in everything from network conditions to fuel prices, simplification of the electrical network is frequently a first step.

Multi-area economic analysis

A typical model for multi-area economic analysis replaces the full AC circuit model with a "ball and stick" model. Every individual generator is assigned to a single zone or "ball" and an aggregate output versus cost function determined for each zone. The "sticks" represent an aggregate network model where each "stick" corresponds to an interface or a collection of transmission elements. Each zone to zone transfer and "stick" is associated with a distribution factor that approximates the percentage of the bilateral transfer between zones that would be carried on the "stick". The combined network and market interaction is then modeled using a linear optimization program for each time period under investigation. The capacities of the "sticks" determine the inter-area constraints which influence the resulting net exports and expected prices in each zone. The goal of the model is to correctly forecast the prices for correctly forecasted initial conditions (demand and generation availability in each zone).

The ability of the model to reflect the actual system depends in part on obtaining reasonable estimates of the capacities of the "sticks" as well as the proper distribution factors for each zone to zone transfer. Both these quantities are reasonably obtained from transfer capability computations.

A first approximation for the capacity of the sticks would be the sum of the ratings of the transmission elements that make up a zone to zone interface. However, since constraints away from the interface can limit interface flows before any of the interface elements are fully loaded, a method that also considers limitation due to non-interface elements is preferred.

By performing a series of transfer capability computations for many combinations of zone to zone transfers, both the distribution factors and capacities can be reasonably approximated. Starting from a reasonable base case with a full network model, one zone is selected as a reference zone. The transfer capability between that zone and every other zone is computed. The distribution factors are obtained from comparison of the base interface flows to those corresponding to a given transfer level. By identifying the transfer capability for many zone to zone transfers, limiting elements that are not interface elements can be identified and incorporated as "sticks" in the ball and stick model. The fundamental step in building the model is the transfer capability computation.

Evaluating economic impacts of transmission expansion

The transfer capability into a region is often used as a consideration when evaluating the expansion of a network. Unlike the previous example, in this case the aggregation and approximation revolves around the economic considerations and a detailed network model is employed for the transfer capability computations. Since the service life of a transmission facility is hopefully very long, a great variety of scenarios might be examined both with and without the candidate facility improvements. The purpose of these studies is usually to demonstrate the increase in transfer capability resulting from an improvement and the corresponding increase in reliability and reduction in cost to serve demand within a region. The computed transfer capability is then used as a measure of the quantity of external generation that can replace internal generation in Loss of Load Probability (LOLP) computations and as an indication of how much cheap power could be transferred into or out of the region through the system.

2.1.3 Transfer capability and electricity markets

The preceding examples described the use of transfer capability for modeling market behavior. A similar application of transfer capability computations influences market operation.

Bilateral and pooled markets are fundamentally different. However, there is some overlap since pooled markets accommodate bilateral trading and the proposed RTOs designed to facilitate pure bilateral markets also plan to offer imbalance services that resemble the location based marginal pricing characteristic of pooled markets. The following examples illustrate uses of transfer capability in the operation of electricity markets.

2.2 Bilateral markets

Currently, transmission providers in the United States implement a reservation system for their transmission systems using the Open Access Same-time Information System (OASIS). Bilateral transactions between parties require a transmission reservation for the transmission systems likely to experience increased flows due to the implied transfer corresponding to the contract. For the purpose of managing the reservation system, transmission providers post the Available Transfer Capability (ATC) for particular area to area transfers that impact their systems.

Energy contracts can span hours or months. Thus, the quantities of transmission available for reservation must be suitable for the range of conditions that can be encountered over those time frames. A portion of the computed total transfer capability (TTC) is reserved and available only on a non-firm basis.

The transfer capability across a transmission system is often used as a basis to determine the quantity of firm transmission service available to schedule energy delivery. For instance, transmission service might be requested a year in advance for a duration of several months. On yet a shorter time frame, the transfer capability across a system is needed to determine the quantity of non-firm transmission service available for an hour or so starting in the next hour or the next day or week. Transfer capability computations are thus integral to the determination of TTC, ATC, and the quantities of transmission reserved for generation and transmission contingencies.

Pooled markets

Within parts of the United States and in Australia and New Zealand, pooled markets operate that do not depend on the computation of transmission capability to establish quantities of transmission available for reservation. However, in these markets, a subtle application of transfer capability computations exists within the context of coordinating auctions for financial rights to congestion rents.

In pooled markets such as PJM, suppliers offer quantities and prices to a pool and load serving entities (LSEs) submit schedules for demand. A central authority determines prices and dispatch based on a constrained optimization program so that all load is served and all network security constraints are observed.

In time intervals when no network constraints actively affect the dispatch, the prices everywhere in the network are nearly the same, subject to differences only due to losses. However, when network constraints are active, such as in the case of a transmission line being fully loaded, prices everywhere can vary significantly depending upon how each location can influence the constraint. Also, by a property of the optimization problem governing the dispatch, during congested conditions the central authority receives more payment from demands than it pays to generators. This quantity of funds is called congestion rent.

Market participants exposed to the variation in prices caused by the discrete nature of constraints desired a mechanism to provide price consistency and hedge against the effects of congestion.

A solution involved the distribution of financial contracts that allowed the holder to receive funds in proportion to the price differences between locations in every hour. These contracts are referred to as "FTRs" for Firm Transmission Rights or Financial Transmission Rights or TCCs, for Transmission Congestion Contracts. Essentially, the congestion rents received by the central authority are distributed to the holders of FTRs.

An interesting problem arises. How can the central authority be sure that they will not sell FTRs that require payments exceeding the congestion rents? The answer [47] involves an optimization program called the simultaneous feasibility test. Each participant wishing to purchase FTRs submits bids identifying a price they are willing to pay for an FTR and the quantity in MW of the FTR and the source and sink locations for which it applies. Note that specification of an FTR abstractly resembles a specification for a physical transfer with the addition of a price per MW. An optimization program maximizes the revenue from the auction subject to the constraint that the total of all FTRs awarded, if implemented as bilateral transfers on the network model, would not violate any network constraints. In other words, the financial contracts are modeled as physical transfers to determine a feasible set of contracts. This is the exact same problem one would solve to achieve an optimum curtailment of physical bilateral transfers to maintain network security.

In these examples, the range of conditions that must be accounted for are substantial and different for each application. The transfer capability can change for different assumptions about demand, the location of transfer sources and sinks, configuration of the network, seasonal and daily changes in facility ratings, variation in dispatch in neighboring regions, and many other factors.

The need to quantify transmission capability requires computations and assumptions. Discussion of the computation is nearly inseparable from discussion of assumptions.

For example, the selection of sources and sinks between the same two areas will be different for a transfer that models the use of reserves to replace a forced generator outage than for a transfer that models the economic activity of market participants. The purpose of the transfer capability analysis determines the assumptions and approximations used for the computations. The challenge and limitation of transfer capability computations is selecting appropriate assumptions.

2.3 Overview of transfer capability computation

The determination of transfer capability for every application typically requires that the transfer capability for a fixed, specific set of assumptions be computed, and then re-computed for some prescribed changes in those assumptions. The purpose of changing the assumptions is to determine the transfer capability most appropriate for the application at hand.

For example, one might desire knowing the most limiting contingency for a particular area to area transfer. In this instance, one would compute the minimum over a set of contingencies of the maximum transfers. Alternatively, one might be interested in the maximum area to area transfer under a fixed network, but varying the point of delivery and point of receipt locations, such as in evaluating the effect the location of reserves has on system security.

In general, analysis of longer and future time frames requires that a greater variety of conditions and assumptions be considered, such as changing load patterns, generator commitments, and network configurations. Analysis of short time frames in near real time has the luxury of more limited conditions but the curse of greater urgency and required accuracy. Computation of transfer capability for one hour ahead might require only analysis of varying branch contingencies subject to a limited set of source and sink assumptions. However, computation of transfer capability over a one week period may require analysis for many different load patterns, generation commitments, and source and sink assumptions. Calculations of Available Transfer Capability (ATC), Capacity Benefit Margin (CBM), and Transfer Reliability Margin (TRM) typically require that the transfer margin computation be repeated for multiple combinations of transfer directions, base case conditions, and contingencies [90], [75].

The basic process involves these steps::

- Establish initial assumptions appropriate to time period of study.
- Compute transfer capability for base assumptions.
- Determine or apply systematic changes to assumptions.
- Recompute transfer capability.

There are many ways of implementing this process. Some of the assumptions that must be specified include:

- Facility ratings.
- Generator commitment and dispatch.

- Demands.
- Source, sink, and loss specifications.
- Power system model and operation (DC/AC, automatic controls, interchange, economic dispatch) and network topology (outages).

The transfer margin computation can be implemented with a range of power system models and computational techniques. One convenient and standard practice is to use a DC power flow model to establish transfer capability limited by thermal limits. The limiting cases are then checked with further AC load flow analysis to detect possibly more limiting voltage constraints.

Alternatively, a detailed AC power system model can be used throughout and the transfer margin determined by successive AC load flow calculations [38] or continuation methods [14, 22, 3, 92]. A related approach [e.g., EPRI's TRACE] uses an optimal power flow where the optimization adjusts controls such as tap and switching variables to maximize the specified transfer subject to the power flow equilibrium and limit constraints. The formulations in [38] and [95] show the close connection between optimization and continuation or successive load flow computation for transfer capability determination.

Methods based on AC power system models are slower than methods using DC load flow models but do allow for consideration of additional system limits and more accurate accounting of the operation guides and control actions that accompany the increasing transfers. Under highly stressed conditions the effects of tap changing, capacitor switching, and generator reactive power limits become significant. A combination of DC and AC methods may be needed to achieve the correct tradeoff between speed and accuracy. The methods in this tutorial account directly for any limits which can be deduced from equilibrium equations such as DC or AC load flow equations or enhanced AC equilibrium models.

2.4 Generic transfer capability

For the purposes of understanding the many methods of transfer capability, it is beneficial to consider the simple case of computing the transfer capability for a situation with a limited set of variable assumptions.

A single transfer capability computation yields:

1. A base case.
2. Specification of the transfer direction including source, sink, and losses.
3. A solved transfer-limited case and a binding security limit. The binding security limit can be a limit on line flow, voltage magnitude, voltage collapse or other operating constraint. Further transfer in the specified direction would increase the violation of the binding limit and compromise system security.
4. The transfer margin is the difference between the transfer at the base case and the limiting case.

Typically, we think of the base case and the transfer specification as inputs to the process and the identification of the limiting case and transfer capability as the outputs. However, circumstances exist for which the transfer specification is also an output. The four components of the computation are now explained in greater detail.

Base Case: A time horizon for the transfer capability is selected and a base case consistent with the time horizon is selected. The time horizon refers to the period in the future for which the transfer capability must be found. For example, to schedule firm transmission service to serve native base load, one is commonly interested in knowing a particular transfer capability for both peak and non-peak periods over an entire month, often three to six months or more in the future.

Identification of a suitable base case is a formidable and important task. In the case of very short time frame analysis, an example of a suitable base case would be the most recent case available from an EMS state estimator updated to include load forecast and schedules for the period of question. Even in this case, several base cases may need to be developed to reflect possible contingencies. Selection of base cases for longer term analysis can require extensive studies. However, such base cases are normally useful for many planning purposes.

Regardless of the time horizon in question, several important considerations are inherent in selection of one or more base cases. The base case assumes a particular set of facility ratings, demands, unit commitment and generator dispatch and thus implies loading on all facilities. Changes in any of these assumptions lead to a different transfer capability.

For the purposes of this example, we assume that a suitable base case has been identified. The base case is assumed to be a secure and solved case.

Specifying the transfer: A transfer is specified by changes in power injections at buses in the network. For example, a point to point transfer from generator A to generator B is specified by increasing power at generator A, reducing power at generator B and making some assumption about where additional power will be generated to cover the losses associated with the transfer. In particular, if 100 MW are to be transferred from A to B, then power at generator B is reduced by 100 MW and power at generator A is increased by 100 MW plus an amount to cover the change in losses. An alternative assumption is that the power at generator B is reduced by 100 MW, power at generator A is increased by 100 MW, and generation at other locations supplies the additional power required by losses. Another way to think about transfers is in terms of changes in exports from areas or between areas in the power system. Although these changes in area exports can summarize the effect of a transfer, they do not completely define a transfer unless the generation dispatch within the areas is also specified (there are many ways to dispatch generation within an area to change the area exports). That is, the participation of each generator in the area in the transfer has to be specified. A complete specification of changes in area exports and generation dispatch within each area is equivalent to specifying the power injections at all generators.

Limiting Case: A solved transfer limited case is established at which the system transfers have been changed and there is a binding security limit. The binding security limit can be a limit on line flow, voltage magnitude, voltage collapse or other operating constraint. Further transfer in the specified direction would cause the violation of the binding limit and compromise system security.

2.5 Continuation methods

One way to compute transfer capability with a software model is called continuation. From the solved base case, power flow solutions are sought for increasing amounts of transfer in the specified direction. The quantity of the transfer is a scalar parameter which can be varied in the model. The amount of transfer is gradually increased from the base case until a binding limit is encountered. This continuation process requires a series of power system solutions to be solved and tested for limits. The transfer capability is the change in the amount of transfer from the base case transfer at the limiting point. Continuation can be simply done as a series of load flow calculations for increasing amounts of transfers. However, when convergence could be poor, such as the case for transfers approaching voltage instability, methods that allow the transfer parameter to become a dependent variable of the model are the most successful.

Some continuation software accounts for power system nonlinearity, operator actions, controls such as tap changes, and application of generator limits as the transfer is increased. These programs are most valuable for transfer computations aimed at assessing system security margins or the effects of very near term (next hour) transactions. The appropriate use of these programs depends on the application.

Thousands of simultaneous transactions make up the energy schedule that the system dispatchers observe. The state of the power system evolves from hour to hour more or less by following the schedule, not by implementing sequences of individual transactions. The manner in which a transfer is applied during the computation might have no relation to how the transfer is implemented in the actual system.

Useful estimates of transfer capabilities can be obtained with simpler power system models such as the DC load flow approximation. A DC model may be preferable to an AC model particularly in circumstances where the extra data for an AC model is unavailable or very uncertain, such as the case of very long time frame analysis. The added detail of an AC model is not useful if it contributes greater uncertainty.

The DC approximation is good for these reasons:

- Fast computation - no iteration.
- Thermal limits, MW limits
- Network topology handled with linear methods.
- Good approximation over large range of conditions
- Minimize data requirements

The DC approximation is poor for these reasons:

- Cannot identify voltage limits
- Is not accurate when VAR flow and voltage deviations are considerable.
- Over use of linear superposition increases errors.

2.6 Optimal power flow approaches

In the transition to a more competitive electric power market place, the past several years have seen considerable interest in tools to determine available transmission capability, to manage congestion when limits are approached or exceeded, and to characterize the marginal value access to capability on a transmission corridor through transmission rights. In the traditional regulated utility environment, the so called Optimal Power Flow (OPF) was the tool of choice for a range of optimization problems relating to real time and near real time operation [62], [63]. However, new objectives in a competitive environment [46], [53], the associated focus on congestion management to facilitate competition [78], and the increasing volatility of dispatch and operating conditions raise significant new challenges in OPF [65], [44].

A number of authors have recognized the possibility of extending OPF tools to the problems of ATC calculation, congestion management, and transmission access valuation. Indeed, the period since 1999 has seen a flurry of activity, as represented by such works as [25], [66], [72], [93], [24], [64], [51], [98], [71]. All of these works share the common theme that they formulate an optimization problem in which the dominant elements are the equality constraints arising from the power flow. To the extent that the term "optimal power flow" has a single definition, the use of network power flow constraints qualifies these treatments as OPF problems. Variations in these works relate to the nature of the inequality constraints represented, ranging from relatively basic operational equipment limits [71] to more detailed formulations that attempt to approximate transient stability security requirements through tractable algebraic inequalities [72]. The objective functions selected in these works also vary widely, though treatment of a range of objective functions has always been a hallmark of the OPF literature (again, the complexity of OPF problems lies in their high dimension network constraints).

Our goal here is to provide a brief representative example of how an OPF problem may be formulated to identify the available transfer capability associated with a transmission corridor. To this end, it is useful to begin from a simple representative example of a security constrained ATC maximization to allow concrete illustration of the form of objective function and constraints. This formulation will be extended to treat uncertainty in Section 6.9.

Let $\mathbf{V}$ represent the complex voltages at each node of the transmission network. These are the basic decision variables of the optimization framework. The key constraints relate to the electrical behavior of the network. Recognizing that the voltage/current behavior is well approximated as linear, the most fundamental fixed parameters of the problem are those that describe the electrical characteristics of the transmission network. In particular, each branch k is parameterized by its complex admittance parameter $\mathbf{y}_k$. Note that realistic problems contain on the order of 10^4 nodes, with an average interconnection density of 3 to 4 branches per node.

For our illustration here, we will assume that a transmission corridor is defined by a set of transmission lines that form a cutset, separating one region of the network from another. Let Γ denote the set of indices corresponding to transmission line branches forming the transmission corridor. Moreover, without loss of generality, we will assume that we have a specified direction of desired power flow across the corridor, and that all lines have their arbitrary reference directions assigned to agree with this direction of flow (in circuit theoretic terms, this reference direction is simply the convention used for measuring branch current, positive flows agree with the reference direction, while negative flows run counter to it). It will prove convenient to characterize the flow through the transmission corridor in terms of the sum of currents over all the lines comprising the corridor. While one may alternatively select active power flow through the corridor as the quantity to maximize, this choice introduces a significantly greater degree of nonlinearity into the objective function, without a commensurate improvement in the practical usefulness of the result.

Inequality constraints to be satisfied in the formulation below include limits on the magnitude of current flow in each line of the network, upper and lower limits on bus voltage magnitude at each bus, and upper and lower limits on active and reactive power at each bus. For simplicity of notation and algorithmic description, it is convenient to represent active and reactive power equality constraints as inequalities in which upper and lower limits are selected to be equal (or, as is often useful for improved algorithmic performance, nearly equal within a small tolerance commensurate with the desired power flow solution accuracy). Likewise, buses at which voltage magnitude is regulated to a setpoint have upper and lower limits on voltage magnitude set equal to this setpoint.

With this notation, our representative optimization formulation of the ATC calculation may be summarized as:

$$\max_{\mathbf{V}} \sum_{k \ in \ \Gamma} \mathbf{i_k} \tag{2.1}$$

subject to

$$\mathbf{I}^{min} < \mathbf{i}(\mathbf{V}) < \mathbf{I}^{max}$$

$$\mathbf{V}^{min} < |\mathbf{V}| < \mathbf{V}^{max}$$

$$\mathbf{P}^{min} < Re\{\mathbf{S}(\mathbf{V})\} < \mathbf{P}^{max}$$

$$\mathbf{Q}^{min} < Im\{\mathbf{S}(\mathbf{V})\} < \mathbf{Q}^{max}$$

where the complex current flow on branches, $\mathbf{i}$, satisfies

$$\mathbf{i}(\mathbf{V}) = \mathbf{y}. * [\mathbf{A}^T\mathbf{V}]$$

and the complex power absorbed into the network from each node, $\mathbf{S}$, is given by

$$\mathbf{S}(\mathbf{V}) = \mathbf{V}. * \mathbf{A}(conj(\mathbf{y}. * [\mathbf{A}^T\mathbf{V}]))$$

where $\mathbf{A}$ is the node-to-branch incidence matrix and the limit thresholds $\mathbf{I}^{min}$, $\mathbf{I}^{max}$, $\mathbf{V}^{min}$, $\mathbf{V}^{max}$, $\mathbf{P}^{min}$, $\mathbf{P}^{max}$, $\mathbf{Q}^{min}$ and $\mathbf{Q}^{max}$ are all given real-valued parameters, as described previously.

In this formulation, one has the advantage that the objective appears as a purely linear function of the underlying decision variables, which are the complex bus voltage magnitudes. Nonlinearity is introduced by the quadratic dependence of complex bus powers, $\mathbf{S}$, on these complex bus voltages. The advantages of treating the complex bus voltage quantities in rectangular coordinates, in order to exploit the resulting quadratic form of the power flow constraints, has been described in many previous works; for example, see [37] and [9].

2.7 Linear methods

The use of linear approximations for updating and estimating transfer capability is widespread. This section introduces common terminology and the fundamental concepts.

Power Transfer Distribution Factors, commonly referred to as PTDFs, express the percentage of a power transfer that flows on a transmission facility. For example, if the component corresponding to the Z transformer of the PTDF for a Bus A to Bus B transfer was 0.5, then a transfer from A to B of 400 MW would result in an increase flow of 200 MW on the Z transformer. The transmission facilities, commonly referred to as flowgates, can be transformers, lines, or sets of transformers and lines. PTDFs are most useful for estimating the change in flows that result from a particular transfer and identifying which flowgates are most affected by that transfer. The sources and sinks for the power transfer must be specified for the PTDFs to be computed. In addition to specification of sources and sinks and definition of flowgates, the computation of PTDFs also depends upon the model of the power system selected. However, most commonly PTDFs are computed with the DC model assumptions.

Shift Factors, commonly referred to as GSFs (Generator Shift Factors) or Adjustment Factors, express the change in flow on a particular flowgate that results from increasing generation at a node. GSFs are meaningful only when considered in source-sink pairs, since power injected at one location must be matched by power removed at another location. When the sink is not identified for a GSF, it can be assumed that the system slack bus has been used as the sink. GSFs are most useful for identifying which generator pairs can influence a particular flowgate. Assumptions that affect the computation and application of PTDFs also affect GSFs. An example of GSF in a DC load flow model is given in section 1.5.

GSFs are very closely related to PTDFs. For example, consider a matrix where each row corresponds to a flowgate and each column corresponds to a generator. Let each element of the matrix represent the GSF for the flowgate corresponding to that row and the generator source corresponding to that column, and a constant sink. Then each row represents the vector of GSFs for one flowgate assuming the constant sink, and each column of the matrix is the vector of PTDFs for all flowgates for the transfer between one generator and the system sink. The distinction between GSFs and PTDFs mostly concerns how one cares to view the same data. For the case of a lossless model, the PTDFs for all possible transactions can be obtained from the matrix of GSFs computed with a constant sink.

NERC provides both PTDF and GSF web based viewers and downloadable data files of PTDFs. The NERC PTDFs are used in determining which transactions are subject to curtailment when NERC Transmission Line-loading Relief (TLR) procedures are invoked on a constrained flowgate. The PTDFs and GSFs that are available from NERC are computed using the lossless, DC load flow model and some other simplifying assumptions. A new tool called the Flowgate Information Study Tool (FIST) will appear in 2001.

Chapter 3

Sensitivity of transfer capability

A variety of applications in both planning and operations require the repetitive computation of transfer capabilities. Transfer capabilities must be quickly computed for various assumptions representing possible future system conditions and then recomputed as assumptions and system conditions change. There is also uncertainty in each transfer capability calculation due to uncertainty in the assumptions and data used. Thus it is useful once a transfer capability has been computed to be able to compute the sensitivity of that transfer capability to data. In practice these sensitivities can be computed very quickly for a wide range of parameters. The sensitivities can be used to estimate the effect on the transfer capability of variation in simultaneous transfers, assumed data, and system controls.

3.1 Explanations of sensitivity

Although sensitivity is a single concept, like most generally useful concepts, it can be thought of and applied in different ways. Sensitivity is a widely known and very widely used concept, but since it is a key concept underlying much of the material in this document, this section gives a tutorial explanation of sensitivity in several ways.

To be specific in the wording of the explanations, we explain the sensitivity of a transfer capability T with respect to a parameter p. In practice, p could be any parameter, including the amount of a real power at a load, the amount of a simultaneous transfer, a control setting, or a value of line impedance. However, for specificity in the wording of the explanations, we assume that p is the real power consumed at load bus 4.

Suppose that at the base case, the real power p consumed at load bus 4 is 70 MW and that the transfer capability T at this base case is 133 MW. We are interested in how much T varies from 133 MW when the bus 4 load p is changed from 70 MW. We can write $\Delta T = T - 133$ and $\Delta p = p - 70$, so that we will be interested in how much ΔT varies when Δp is changed.

T is a function of p. The sensitivity of T with respect to p is the derivative of T

with respect to p:

$$\text{Sensitivity of } T \text{ with respect to } p \ = \frac{dT}{dp} \tag{3.1}$$

It follows from calculus that if Δp is small, then it is approximately true that

$$\frac{\Delta T}{\Delta p} = \frac{dT}{dp} \tag{3.2}$$

and

$$\Delta T = \frac{dT}{dp}\Delta p \tag{3.3}$$

and the approximations (3.2) and (3.3) become exact as Δp becomes vanishingly small. In our specific numerical example the sensitivity $dT/dp = -0.4$ so that the approximation becomes

$$\Delta T = -0.4 \ \Delta p \tag{3.4}$$

That is, if p increases by 1 MW, then T decreases by 0.4 MW. Equivalent to (3.4) is

$$T = 133 - 0.4 \ (p - 70) \tag{3.5}$$

which is a linear approximation for how T depends on p for p near 70 MW.

Indeed, one of the most useful approaches to sensitivity is as follows: We know that T is a nonlinear function of p. For p near 70 MW we approximate this nonlinear function by the linear function (3.5). The sensitivity is the coefficient -0.4 of p in (3.5) or the coefficient -0.4 of Δp in (3.4).

If we instead work near another value of p such as $p = 60$ MW, the sensitivity coefficient will change in value.

The illustrative numbers used above can be checked for the transfer from bus 1 to bus 5 at the base case of the 6 bus system on the calculator available at

http://www.pserc.cornell.edu/tcc/

Simply press CALCULATE and then compute the quick estimate of the transfer capability with the parameter change of load at bus 4 +10MW. The quick estimate for transfer capability evaluates and uses the sensitivity for the computation. In particular, the quick estimate calculates the sensitivity -0.4 of the transfer capability with respect to the load at bus 4 and then estimates the transfer capability T when the load at bus 4 is 80=70+10 MW using (3.5) to get

$$T = 133 - 0.4 \ (80 - 70) = 129 \text{ MW} \tag{3.6}$$

Moreover, the calculator shows the sensitivity graphically. In particular, the sensitivity determines the slope of the gray line (the slope of the gray line is $1/\textit{sensitivity}$, or $1/(-0.4) = -2.5$ in our illustrative numbers.)

The operation of the calculator should be thought of as follows: The AC continuation power flow is run to find the transfer capability for the selected base case. Then the quick estimate calculates the sensitivity of this transfer capability to the chosen parameter. The chosen parameter determines the vertical axis of the graph and the sensitivity determines the slope of the gray line passing through the calculated transfer capability. The gray line is a linear model for changes in the transfer capability when the chosen parameter is varied. The calculator then uses the gray line to estimate the transfer capability for the desired parameter change according to (3.5).

3.2 Sensitivities in DC load flow

Suppose that a linearized (DC load flow) model for the entire power system is chosen. In this special case, the relation between the transfer capability and the load at bus 4 is linear and the equations (3.2) and (3.3) are exact. Suppose that the event limiting transfer capability was the maximum flow limit on the transmission line 1-5. Then for linearized (DC load flow) model, the sensitivity of the transfer capability with respect to the load at bus 4 is the negative of the generation shift factor for bus 4 and line 1-5. (Suppose we increase the load at bus 4 by 1 MW. Then the flow on line 1-5 increases by an amount given by the generation shift factor for bus 4 and line 1-5. Since this amount of increase in flow on line 1-5 will decrease the transfer capability by the same amount, this amount is also the negative of the sensitivity of the transfer capability with respect to the load at bus 4.)

Since the sensitivities are equal to the negative of the generation shift factors in the case of flow limits in a DC load flow model, we see that transfer capability sensitivities for an AC load flow model generalize the well known generation shift factors to the AC case. The transfer capabilities calculated for an AC loadflow take account of power system nonlinearity and can handle voltage magnitude and voltage collapse limits. Moreover, the AC load flow model has parameters available such as voltage magnitude and reactive power loads which are not accounted for in the DC load flow model. The variations (and hence the sensitivities) of the transfer capability with respect to voltage magnitude and reactive power loads are of interest.

3.3 Estimating interactions between transfers

One concern is how a transfer capability T varies if another transfer S is varied from its base case value. The situation can be shown graphically by a curve showing how the transfer capability T varies as the transfer S is varied. The transfer capability T at the base case of transfer S is the length of the horizontal line shown in Figure 3.1. The curve shows conceptually how the two transfers interact. With repetitive calculation many points on the curve can be exactly calculated. However, in practice, when time is short, it is useful to be able to approximate the curve by a tangent line. The tangent line approximation at the base case of transfer S is shown in Figure 3.1.

The reciprocal of the slope of this tangent line approximation is the sensitivity of the transfer capability T to transfer S. That is, the slope of the tangent line approximates how the two transfers interact. The slope of the tangent line in Fig-

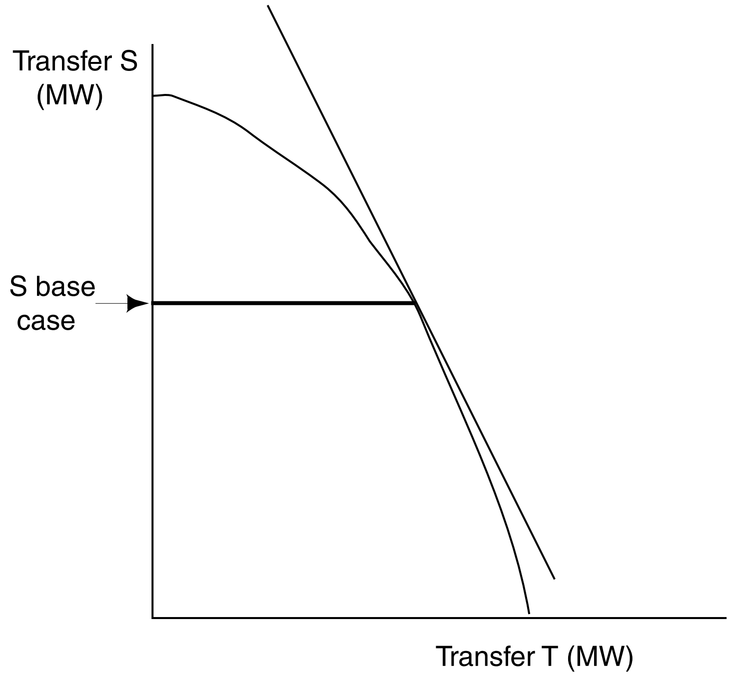

Figure 3.1: Effect of transfer S on transfer T.

ure 3.1 is -2; adding 1 MW to transfer S decreases transfer capability T by 0.5 MW. Thus the sensitivity of transfer capability T with respect to transfer S is -0.5.

For two transfers, we obtain an area of feasible transfers which is bounded by curves which are the combinations of the two transfers which correspond to transfer capabilities. Given a point on the boundary curves, we can estimate the nearby boundary using sensitivities to define the tangent lines to the boundary.

This generalizes to many transfers interacting: we obtain a hypervolume of feasible transfers which is bounded by curved hypersurfaces which are combinations of all the transfers which correspond to transfer capabilities. Given a point on the boundary hypersurfaces, we can estimate the nearby boundary using sensitivities to define the tangent hyperplanes to the boundary. An interpretation of the hyperplanes is that they represent trade-offs or interactions between the power transfers at the limiting case.

3.4 Fast formula for sensitivity and 3357 bus example

This section is in the following paper:

Sensitivity of transfer capability margins with a fast formula

This paper is reprinted at the end of the document.

(This paper is also reference [2].)

Chapter 4

Applications

4.1 Available transfer capability

Successful implementation of electric power deregulation requires the determination of the available transfer capability of a power system. The available transfer capability indicates the amount by which interarea bulk power transfers can be increased without compromising system security.

The value used for available transfer capability affects both system security and the profits made in bulk power transactions. Moreover, market participants can have conflicting interests in a higher or a lower available transfer capability. Thus, under deregulation, there is increasing motivation for defensible calculations of available transfer capability and its components such as transmission reliability margin.

For this section we use the main features of the NERC 1995 and 1996 definitions [89, 90]: The power system is judged to be secure for the purpose of interarea transfer if "all facility loadings are within normal ratings and all voltages are within normal limits", the system "remains stable following a disturbance that results in the loss of any single element", the post-contingency system ... has all facility loadings within emergency ratings and all voltages within emergency limits" [74].

The time horizon of the calculation is established and a secure base case is chosen. A base case transfer including existing transmission commitments is chosen. Then a transfer limited case is determined. One method to determine the transfer limited case gradually increases the transfer starting at the base case transfer until the first security violation is encountered. The real power transfer at the first security violation is the total transfer capability.

The available transfer capability is then defined as

$$\begin{aligned}&\text{Available Transfer Capability (ATC)} = \\ &\quad \text{Total Transfer Capability (TTC)} \\ &\quad -\text{Existing Transmission Commitments (ETC)} \\ &\quad -\text{Transmission Reliability Margin (TRM)} \\ &\quad -\text{Capacity Benefit Margin (CBM)}\end{aligned} \tag{4.1}$$

The calculation may be repeated for a short list of contingencies and the minimum of these available transfer capabilities is used.

For further summary and explanation of ATC see [74, 89, 90].

4.2 The economics of power markets and the Poolco model

This section gives a brief background on the economics of power markets.

Electricity restructuring was driven in large part by the realization that new technologies were making market competition in electricity generation feasible. However, it was also recognized that the transportation of electricity would be still considered to be a natural monopoly service and continue to be regulated. The market design for electricity markets therefore involved attempting to find a model that would facilitate competition in generation while still keeping transportation services under regulatory supervision. To realize the full benefits of competition in generation services, it was recognized that full non-discriminatory open-access to transmission by all market participants was necessary. Thus the market design for efficient electricity markets revolved around finding the best way for the regulated transmission company ("transco") to give access to generator companies ("gencos") to achieve the most efficient generator dispatch possible.

Any market design for electricity must take account of the special features of electricity that differentiate it from other commodities.

- Electricity cannot be stored and must be supplied the instant it is demanded. Since this is a physical constraint that must be respected at all times (or else blackouts will ensue), the power system must be operated by a system operator at all times for reliability purposes.

- Power flows in networks follow Kirchhoff's laws of physics, and cannot be directly controlled. For example, when a trader injects electric power in one location of the network and takes it out at a different location, the trader generally has no control of the way in which the power distributes itself among the various transmission lines. In other words, it is impossible for a trader to specify the route that the electric power follows. This is in contrast with other commodity markets, where the shipper can generally choose the transportation routes over which to ship the commodities.

Therefore, to optimally resolve transmission congestion, an Independent System Operator (ISO), who is a regulated entity, and who is acting on behalf of the transco, must arrange for trades in real time among market participants to achieve the most efficient, i.e., least cost, dispatch. This is the *poolco* model. In this model, traders shipping power offer the ISO "buy" or "sell" prices at which they will trade power at any node. The ISO uses this information to dispatch the system resources at least cost using an Optimal Power Flow while taking account of transmission congestion. This process results in *efficient* spot prices that generally vary by location. The prices are efficient in the sense that the system demand is met by a least-cost combination of supply, while still respecting transmission constraints; in the sense that all "buy"/"sell" trades that are bid/offered above/below the resulting nodal market clearing prices are accepted and are charged/paid their respective nodal market clearing prices; and in the sense that those who place the highest values on (congested) transmission capacity are awarded rights to this capacity. For example, a generator that offers $20/MWh at a node that clears at $25/MWh will be accepted and be awarded $25/MWh for all of its offered output. The theory of nodal pricing of electricity originated with the MIT Energy Laboratory and appeared in Caramanis, Bohn, Schweppe (1982); Bohn, Caramanis, and Schweppe (1984); and Schweppe, Caramanis, Tabors, and Bohn (1987)[16, 13, 76]. In the last decade, the locational pricing of electricity has been popularized and strongly advocated by Hogan [47, 48, 49], who also devised the concept of a *Transmission Congestion Contract* as a way of managing or hedging transmission risk (for future periods) between any two locations of an electric power system. Other useful references that deal with hedging transmission risk are Stoft (1998) [81] who suggested having a specific kind of electricity futures markets based on the Chao-Peck congestion pricing scheme [20]; and Rajaraman and Alvarado (1998b) [70] that generalized Stoft's result, and developed the theory and procedures for managing transmission risk and detecting locational arbitrage opportunities using liquid futures markets in a poolco setting. Systems where the poolco model are functional (more or less) include Pennsylvania-Jersey-Maryland (PJM) system, the New York electric system, the New Zealand electric system, and the Australian electric system.

An alternative to the poolco system, but that theoretically could reach the same efficient solution as the poolco model, is due to [96, 20, 21]. In this model, whenever flows in transmission lines violate their flow limits, the ISO makes arbitrary curtailments in trades (without necessarily looking at economical curtailments) to resolve this congestion. It can be shown that in a competitive market, the market participants would then trade among themselves to reach the same solution as the poolco solution. The obvious practical problem with this approach is that market participants would need a great deal of real time information about the transmission system to trade among themselves to reach the efficient solution (whereas in the poolco model, the ISO would do these trades based on "buy" and "sell" bids). With the present day information technologies this method appears to be impractical for clearing markets in real time.

Yet another alternative to the poolco system is NERC's Transmission Line Loading Relief (TLR) system (and other variants of this approach). This is the most inefficient way of resolving transmission congestion. In this method, a region's security coordinators would simply curtail all bilateral trades to relieve transmission congestion in an ad hoc manner using an administrative formula. The formula would not take any account of the willingness of market participants to pay for shipping electricity across congested transmission paths. For more details about why NERC's TLR procedures are inefficient, how they result in economic waste, and how they could potentially lead to anti-competitive gaming, see Rajaraman and Alvarado (1998) [70].

The success of the PJM model has demonstrated that the poolco approach is not only theoretically — but also practically — superior to administrative methods (such as NERC's TLR protocols) for resolving transmission congestion.

4.3 Nodal prices/Poolco

This section shows how the power transfer capability concepts presented in this document can be applied by an ISO running a poolco system.

We briefly digress to explain the role of an ISO in a poolco model. The ISO has the following main functions:

1. Resolve transmission congestion at least cost in the spot market.
2. Ensure that energy supply always equals energy demand (including losses) in real time.
3. Ensure transmission system security at all times.
4. Communicate effectively with market participants about the transmission system in order to facilitate efficient trades.

For example, as part of their functions, an ISO such as the PJM ISO or the NYISO would typically run several Optimal Power Flow solutions per hour; they would broadcast available transmission capacity indices on key transmission paths to market participants; they would coordinate outages of transmission facilities with the transmission owners to ensure system security, etc..

We now give examples of how the power transfer capability concepts explained in this report can be used by the ISO in a poolco setting.

- The ISO can use the formulas in section 3.4 to measure the sensitivity of loadings on different transmission lines to different parameters. For example, the ISO may wish to publish Available Transmission Capacity for the next day for some key transmission paths, given the load forecast. The ISO could simply measure the ATCs for one particular generation configuration and then use the sensitivity formulas to estimate the change in ATC for different generation configurations. Based on ISO experience of generator outages and generation dispatch pattern, the ISO could then publish appropriate (e.g., conservative) ATC margins based on these sensitivity calculations.
- The ISO may use sensitivity formulas to detect the potential for market power. One example of market power is that of a market participant who deliberately withholds generation capacity in one location to cause congestion in one or more transmission lines to manipulate nodal prices. Such kinds of behavior can be detected by estimating the sensitivity of the loading margin on these transmission lines to changes in generator output at all locations. The bidding behavior of those market participants that own generation in those locations that have the most significant impact on transmission line flows could be subjected to a higher level of monitoring.

- Suppose that transmission providers are deciding to take down one or more transmission lines for maintenance and would like the ISO's guidance on which time periods offer the highest margin for safety. The ISO could use sensitivity of the worst-case loading margins to key parameters such as load levels to rank the best periods for doing the maintenance work.

- Suppose that the ISO would like to know the locations where operating reserves should be available for some particular cases of generator contingencies. The ISO could use sensitivity of the worst-case loading margins to the generator injections at each node, and rank them by location. Based on these sensitivity calculation (and on other factors such as generator costs), the ISO could then decide on the optimal locations for having generation reserves available.

- Suppose that the ISO is faced with voltage collapse situation on a hot summer day; suppose that the ISO determines that it has run out of options for changing the existing generation dispatch (or transmission configuration), and that load shedding is the only option. Instead of indiscriminate load shedding, the ISO can use the sensitivity formulas in this report to minimize the costs of the blackouts by selecting the most cost-effective locations that provide voltage collapse relief.

4.4 Planning

This section shows how one may use the sensitivity formulas in this report to make generator and transmission investments in a market that operates under the poolco rubric.

In the past, the regulated utility company was obliged to undertake generator and transmission investments to meet the growing load forecast; the regulated utility would be guaranteed a reasonable rate of return if their network expansion proposal passed regulatory muster. Under deregulation, the incentives for generator and transmission investments are different. The problem is different for the regulated transco and the unregulated gencos.

The necessary and sufficient condition for a market participant to invest in a new generator at some location is that the return on this investment is higher than on all other alternative investments (at least as perceived by the investor). (If merchant transmission were possible, the same criteria would be used to evaluate transmission investments also.)

A regulated entity such as a transco could invest in new or old (i.e., upgrade) transmission facilities if the net "benefits" from doing so are positive. The term "benefits" is deliberately left nebulous here and is dependent on many parameters such as regulatory policies, the transco's performance incentives for making the investment, etc. Alternatively, the concept of transmission congestion contracts (TCC) could be expanded to induce appropriate transmission expansion by unregulated market participants [50]. Hogan shows how market-friendly mechanisms can be developed to enable market participants (rather than the transco) to bear the costs of market transmission expansion. It must be mentioned though that the problem could be complicated by network externalities and the problem of "free-riding" (i.e., other market participants could benefit by one market participant's investment decisions without bearing the costs of the investment). Another complication is that transmission investments (and some generator investments) tend to be "lumpy" because of economies of scale; these can complicate the analysis of these investments.

It is beyond the scope of this document to examine what the regulatory policies ought to be to induce optimal investments for transmission or generation, or to examine what the optimum methods for making these investments should be. We will limit ourselves to showing how the methods of this document can be used, in part, in the evaluation of investment decisions in generation and transmission facilities.

As a practical matter, someone planning to make generator or transmission investments needs to know the answers to at least two important problems. One, given that such investments have a large lifetime (20 to 50 years), and that projecting revenues (and costs) over this time-frame is generally a difficult proposition because of inherent uncertainty involved, how should one evaluate the investments? Two, which locations are the optimal places for investing in new generation/transmission?

A partial answer to the first problem is that the investor must have a reasonably good idea of the revenue/cost flow in the first few years, and must rely more on qualitative analysis (or at least approximate quantitative analysis) for the later years. This is where the sensitivity formulas of our report could be useful. For example, an investor locating a new generator may estimate sensitivities of flows along key transmission paths (along which the new generator's output would tend to flow) to changes in various parameters, such as load growth in various load centers, new generator investments by competitors at other locations, changes in transmission configuration, changes in economic dispatch patterns because of changes in fuel prices, etc. These sensitivity estimates could enable the investor to develop — in a computationally efficient manner — a rank of real-life scenarios that may affect the investor's profits; the investor may then analyze some of these scenarios in more detail using more detailed tools.

For the second problem, the sensitivity formulas could aid in the following way. Consider a market participant that is trying to find an optimal location to invest in new generation. Given different candidate locations, the market participant can estimate the sensitivities of key variables such as transmission line flows, transmission congestion pattern, loading margins to operational limits, etc., to changes in MW injections at the candidate locations. The sensitivity results can then be used to rank the different locations appropriately; more detailed analysis can then be performed on a relatively smaller set of cases that have the highest sensitivity to the investment. For example, if transmission congestion on transmission paths connecting the generator to the load-center is increased as a result of generator investment in one location relative to other locations, then investment in that location is a less desirable because the generator may be constrained off under congested conditions.

In summary, sensitivity analysis can be a very useful computational aid in solving complex generator or transmission investment problems. The primary purpose of the sensitivities is to be used as a screening (and ranking) tool to get (even if approximate) estimates of the value of the investment under different "what-if" scenarios. These sensitivity results can provide guidance to conduct more detailed analysis on a few targeted set of scenarios or investment possibilities.

4.5 Market redispatch

This section is in the following paper:

Interactions among limits during maximum loadability and transfer capability determination

This paper is reprinted at the end of the document.

(This paper is also reference [1].)

4.6 Summary of paper by Corniere et al.

Available Transfer Capability in a market-oriented context and in an environment where uncertainty plays a major role requires careful balancing of exactly what is meant by "maximum" and "available." In a recent work [25] the authors have done an excellent job in discussing and defining issues of measuring and monitoring ATC under these conditions. The transfer capability is determined by increasing the studied transfers according to an incremental step and to assumptions on power injections until congestion cannot be relieved by any form of redispatching. This transfer level is referred to as the Maximum Transfer Capability (MTC). For each level of power transferred, the paper uses a two-phase approach that permits the determination of the cost upon the system imposed by congestion cost for a given state of the system. The paper provides for an explicit determination of risk for each value of the transfer capability and for each transaction. Risk is defined in this paper as the probability of curtailment of a transaction. Firmness and risk are recognized as two different concepts. Risk assessment takes into account the firmness associated with the transaction, i.e. the conditions defined for its curtailment (maximum congestion cost accepted for the transaction and priority among simultaneous transactions). Risk for a transfer is determined as the proportion of simulated situations where the desired transfer level cannot been reached. The paper illustrates the importance of considering redispatching in the determination of additional transfer limits for highly stressed systems.

4.7 Background survey of security and optimization

This section reviews the main elements of power system security and optimization as they pertain to transfer capability.

Of greatest interest is the notion of maximum loadability and maximum transfer capability. Open access to transmission services requires continual determination of the available transfer capability (ATC) for each interface of a region or area of the power network. The following are the main limits to transfer capability:

- power flow or current limits (normal and emergency), possibly leading to cascading line outages and system separation
- voltage magnitude upper and lower limits (normal and emergency)
- voltage collapse and/or critical generator VAR limits
- transient stability limits

A recent concise review of ATC computations and current terminology, along with a comprehensive bibliography is contained in [74]. Foundational work regarding transmission transfer capability determination is represented in [30, 34, 56, 57, 80], and, concerning sensitivity and optimal power flow, in [82, 67, 29]. NERC documents [89, 90] contain useful terminology and descriptions. The most current information can be found at the OASIS web sites for each operating region. Fundamental work leading to the characterization of limits in probabilistic terms and the translation of these limits to costs can be found in [8, 6, 59].

Continuation methods for the solution of general nonlinear equations and bifurcation problems are described in [33, 77], and the application to power systems is covered in [14, 22]. In particular, [68, 91] illustrate the combined use of continuation methods and sensitivity analysis for security margin computation.

Also of importance is recent work on sensitivity of margins. [41] presents exact analytical margin sensitivity formulas with illustrative examples including the effect of variation in interarea transfers on the loading margin to voltage collapse. [95] applies sensitivity analysis to the determination of the contingencies that most limit transfer capability, and motivates the work on contingency analysis presented in [42].

For a recent survey of the economic theory of electricity pricing, see [27]. Spot pricing of electricity was first proposed by William Vickrey in 1971 and was subsequently elaborated by the MIT Energy Laboratory and summarized in [76]. The theory of locational pricing of electricity was first proposed by R.E. Bohn, M.C. Caramanis, and F.C. Schweppe in 1984 and has subsequently been developed by W.W. Hogan in [47] and its sequels.

The background material relating to optimization and optimum power flows is now summarized. Economic operation of power systems has been a topic that has received much attention since the 1930's. In the late 1950's, Leon Kirchmeyer published his important work on economic operation of a system considering network losses. Subsequently, Carpentier presented a formulation of the Optimum Power Flow problem in 1962, and nicely summarized this work in 1979 [17]. Since this time, much significant work has taken place in this direction. Important early practical references included [29] and [73]. Subsequent, significant progress was attained with the development of quadratic Newton-type optimization methods [83, 87, 88]. For more recent surveys and development in the OPF area, refer to [52]. A recent reference that adds a lot of insight into the workings of the optimum power flow in practical problems is [4].

Chapter 5

Quantifying transmission reliability margin

Transmission reliability margin accounts for uncertainties related to power transmission in transfer capability calculations. This chapter suggests a formula that quantifies transmission reliability margin based on transfer capability sensitivities and a probabilistic characterization of the various uncertainties. The formula is tested on 8 and 118 bus systems by comparison with Monte Carlo simulation. The formula contributes to more accurate and defensible transfer capability calculations.

5.1 TRM and ATC

The available transfer capability indicates the amount by which interarea bulk power transfers can be increased without compromising system security. Available transfer capability is described in more detail in section 4.1. The available transfer capability is the total transfer capability minus the base case transfer together with adjustments such as the Transmission Reliability Margin (TRM), see (4.1).

The transmission reliability margin accounts for uncertainties in the transmission system and safety margins. According to NERC [90], "The determination of ATC must accommodate reasonable uncertainties in system conditions and provide operating flexibility to ensure the secure operation of the interconnected network". There are two margins defined to allow for this uncertainty: The transmission reliability margin is defined in [90] as "that amount of transmission capability necessary to ensure that the interconnected transmission network is secure under a reasonable range of uncertainties in system conditions". The capacity benefit margin ensures access to generation from interconnected systems to meet generation requirements. The capacity benefit margin is calculated separately from the transmission reliability margin. Since uncertainty increases as conditions are predicted further into the future, the transmission reliability margin will generally increase when it is calculated for times further into the future.

This chapter suggests a straightforward method to quantify transmission reliability margin. The method exploits formulas for the first order sensitivity of transfer capability [43, 40]. These formulas can be quickly and easily computed when the transfer capability is determined. The formulas essentially determine a linear model for changes in transfer capability in terms of changes in any of the power system parameters. This chapter supposes that the uncertainty of the parameters can be estimated or measured and shows how to estimate the corresponding uncertainty in the transfer capability. A formula for transmission reliability margin is then developed based on the uncertainty in the transfer capability and the degree of safety required.

In our framework [2], the following limits are accounted for in the transfer capability computation:

- power flow or current limits (normal and emergency)
- voltage magnitude upper and lower limits (normal and emergency)
- voltage collapse limit

Our framework accounts directly only for limits which can be deduced from static model equations. Oscillation and transient stability limits are assumed to be studied offline and converted to surrogate power flow limits.

5.2 Quantifying TRM with a formula

Parameters and their uncertainty

The transfer capability is a function A of many parameters $p_1, p_2, ..., p_m$:

$$\text{transfer capability} = A(p_1, p_2, ..., p_m) \tag{5.1}$$

Uncertainty in the parameters p_i causes uncertainty in the available transfer capability and it is assumed that this uncertainty in the available transfer capability is the uncertainty to be quantified in the transmission reliability margin. The parameters p_i can include factors such as generation dispatch, customer demand, system parameters and system topology. The parameters are assumed to satisfy the following conditions:

1. Each parameter p_i is a random variable with known mean $\mu(p_i)$ and known variance $\sigma^2(p_i)$. These statistics are obtained from the historical record, statistical analysis and engineering judgment.
2. The parameters are statistically independent. This assumption is a constraint that can be met in practice by careful selection of the parameters [18].

Transfer capability sensitivity

We assume that the nominal transfer capacity has been calculated when all the parameters are at their mean values. The uncertainty U in the available transfer capability due to the uncertainty in all the parameters is:

$$U = A(p_1, p_2, ..., p_m) - A(\mu(p_1), \mu(p_2), ..., \mu(p_m)) \tag{5.2}$$

The mean value of the uncertainty is zero:

$$\mu(U) = 0 \tag{5.3}$$

Approximating the changes in available transfer capability linearly in (5.2) gives

$$U = \sum_{i=1}^{m} \frac{\partial A}{\partial p_i} (p_i - \mu(p_i)) \tag{5.4}$$

$\frac{\partial A}{\partial p_i}$ is the small signal sensitivity of the transfer capability to the parameter p_i evaluated at the nominal transfer capability.

When the available transfer capability is limited by voltage magnitude or thermal limits, the sensitivity of the available transfer capability to parameters can be computed using the formulas of [2, 43, 40]. When the available transfer capability is limited by voltage collapse, the sensitivity of the available transfer capability to parameters can be computed using the formulas of [41]. (Topology changes can also be accommodated with limited accuracy using the fast contingency ranking techniques in [42].)

In each case a static, nonlinear power system model is used to evaluate the sensitivities. The computation of $\frac{\partial A}{\partial p_i}$ is very fast and the additional computational effort to compute $\frac{\partial A}{\partial p_i}$ for many parameters p_i is very small [41, 2, 43, 40]. For example, the sensitivity of the available transfer capability to all the line admittances can be calculated in less time than one load flow in large power system models [41, 2].

Approximate normality of U

Since the parameters are assumed to be independent,

$$\sigma^2(U) = \sum_{i=1}^{m} \sigma^2 \left(\frac{\partial A}{\partial p_i} (p_i - \mu(p_i)) \right) \tag{5.5}$$

$$= \sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i} \right)^2 \sigma^2(p_i) \tag{5.6}$$

and the standard deviation of U is

$$\sigma(U) = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i} \right)^2 \sigma^2(p_i)} \tag{5.7}$$

The central limit theorem asserts that (under suitable conditions which are discussed in the appendix) the sum of n independent random variables has an approximately normal distribution when n is large. Reference [45] states: “in practical cases, more often than not, $n = 10$ is a reasonably large number, while $n = 25$ is effectively infinite.” Hence for practical power system problems with many parameters, we expect that the uncertainty U is approximately a normal random variable with mean zero and standard deviation given by (5.7). This approximation gives a basis on which to calculate the transmission reliability margin. The conditions described in the appendix are mild and require little knowledge of the distribution of the parameters.

There are cases in which the central limit theorem approximation does not work so well: As stated in [45], "the separate random variables comprising the sum should not have too disparate variances: for example, in terms of variance none of them should be comparable with the sum of the rest." This can occur in the sum (5.4) when there are a few parameters which heavily influence the available transfer capability (large $\frac{\partial A}{\partial p_i}$) and the other parameters have little influence on the available transfer capability (small $\frac{\partial A}{\partial p_i}$) and are insufficiently numerous. In these cases, accurate answers can be obtained by using the central limit theorem to estimate the combined effect of the numerous parameters of little influence as a normal random variable and then finding the distribution of U with the few influential parameters by Monte Carlo or other means (c.f. [19] in the context of probabilistic transfer capacity). This partial use of the central limit approximation dramatically reduces the dimension of the problem and the computational expense of solving it. For other methods not relying on the central limit theorem see chapter 6.

In all cases the central limit theorem approximation improves as the number of similar parameters increases and thus the approximation generally improves as the power system models become larger and more practical.

Formula for TRM

We want to define the transmission reliability margin large enough so that it accounts for the uncertainty in U with rare exceptions. More precisely, we want

$$\text{probability}\{-U < \text{TRM}\} = P \tag{5.8}$$

where P is a given high probability. This can be achieved by choosing the transmission reliability margin to be a certain number K of standard deviations of U:

$$\text{TRM} = K\sigma(U) \tag{5.9}$$

K is chosen so that the probability that the normal random variable of mean zero and standard deviation 1 is less than K is P. (That is, $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{K} e^{-t^2/2}dt = P$.) It is straightforward to calculate K from P by consulting tables of the cumulative distribution function of a normal random variable [12]. For example, if it is decided that the transmission reliability margin should exceed the uncertainty $-U$ with probability P=95%, then $K = 1.65$. (That is, a normal random variable is less than 1.65 standard deviations greater than the mean 95% of the time.) If it is decided that the transmission reliability margin should exceed the uncertainty $-U$ with probability P=99%, then $K = 2.33$.

Combining (5.7) and (5.9) yields a formula for transmission reliability margin:

$$\text{TRM} = K\sqrt{\sum_{i=1}^{m}\left(\frac{\partial A}{\partial p_i}\right)^2 \sigma^2(p_i)} \tag{5.10}$$

In order to use formula (5.10) we need:

- A choice of uncertainty parameters $p_1, p_2, ..., p_m$ satisfying the three conditions above.
- The variance $\sigma^2(p_i)$ of each parameter.
- Calculation of the sensitivity $\frac{\partial A}{\partial p_i}$ of the transfer capability to each parameter p_i.

5.3 Sources of uncertainty

The available transfer capability is computed from a base case constructed from system information available at a given time. There is some uncertainty or inaccuracy in this computation. There is additional uncertainty for future available transfer capabilities because the available transfer capability computed at the base case does not reflect evolving system conditions or operating actions. These two classes of uncertainty are detailed in the following two subsections.

Uncertainty in base case ATC

- inaccurate or incorrect network parameters
- effects neglected in the data (e.g. the effect of ambient temperature on line loading limits)
- approximations in ATC computation

Uncertainty due to evolving conditions

These uncertainties generally increase when longer time frames are considered.

- ambient temperature and humidity (contributes to loading) and weather
- load changes not caused by temperature
- changes in network parameters
- change in dispatch
- topology changes. This is often referred to as "contingencies." The probabilities of these contingencies can be estimated.
- changes in scheduled transactions

While some of these uncertainties may be quite hard to characterize a priori, it is important to note that it would be practical to collect empirical data on the changes in base cases as time progresses. Then variances of the uncertain parameters corresponding to various time frames could be estimated.

It is important to satisfy the statistical independence assumption when modeling the parameter uncertainty. For example, if the uncertainty of different loads has a common temperature component, then this temperature component should be a single parameter and the load variations should be modeled as a function of temperature (see section 6.1).

5.4 Simulation test results

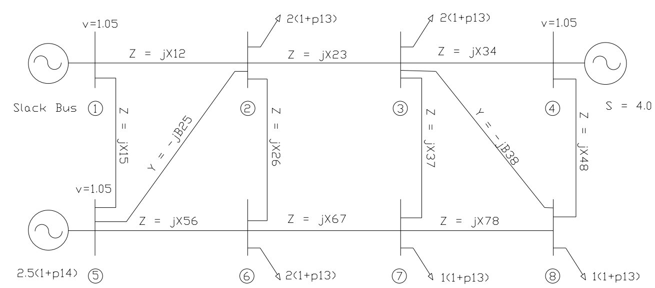

Figure 5.1: 8 bus test system

This section tests the transmission reliability margin formula by comparing it with Monte Carlo simulations in two examples. The first example uses the 8 bus system shown in Figure 5.1. The transfer capability from area 1 (buses 1,2,5,6) to area 2 (buses 3,4,7,8) is limited by the power flow limit on the line between bus 2 and 3. The parameters are listed in Table 5.1. The base case of the system assumes all parameters at their mean values. At the base system, the available transfer capability (ATC) is 2.8253 (with no contingency). Sensitivity of ATC to these parameters can be calculated with no difficulty. Given a desired high probability P, the transmission reliability margin defined in (5.8) is calculated using formula (5.10). Table 5.2 lists transmission reliability margins with respect to different probabilities P. 10,000 samples are used in the Monte Carlo simulation.

The second example uses the IEEE 118 bus system. There are 186 lines and the real power flow limit was assumed to be 1.0 p.u. at all lines except that the real power flow limit for line 54 was assumed to be 3.0 p.u.. We consider a point to point power transfer from bus 6 to bus 45. The uncertain parameters are the power injections to all buses. The power injections are assumed to have a uniform distribution around 5% of their nominal values.

An AC power flow model was used. At the base case, the available transfer capability is 1.8821 p.u.. Given a desired probability P, transmission reliability margin defined in (5.8) is calculated using formula (5.10). Table 5.3 lists transmission reliability margins with respect to different probabilities P. 10,000 samples were used in the Monte Carlo simulation.

In both the 8 and 118 bus examples, the Monte Carlo results confirm the TRM estimates from formula (5.10).

Table 5.1: Parameter distributions

parameter	distribution
line susceptance B25	binary; prob{B25=5.0}=0.95, prob{B25=0}=0.05
line susceptance B38	binary; prob{B38=2.5}=0.95, prob{B38=0}=0.05
line impedance X12	uniform; μ=0.1, σ=0.0029
line impedance X23	uniform; μ=0.2, σ=0.0058
line impedance X34	uniform; μ=0.1, σ=0.0029
line impedance X15	uniform; μ=0.1, σ=0.0029
line impedance X26	uniform; μ=0.1, σ=0.0029
line impedance X37	uniform; μ=0.1, σ=0.0029
line impedance X48	uniform; μ=0.1, σ=0.0029
line impedance X56	uniform; μ=0.1, σ=0.0029
line impedance X67	uniform; μ=0.2, σ=0.0058
line impedance X78	uniform; μ=0.1, σ=0.0029
system loading p13	normal; μ=0.0, σ=0.1
bus 5 generation p14	normal; μ=0.0, σ=0.1
line 2-4 flow limit	normal; μ=1.5, σ=0.1
line 6-7 flow limit	normal; μ=1.5, σ=0.1

Table 5.2: TRM for 8 bus system

P	90%	95%	99%	99.5%
TRM formula	0.6012	0.7750	1.0944	1.2118
Monte Carlo	0.6027	0.7846	1.1083	1.2171

Table 5.3: TRM for 118 bus system

P	90%	95%	99%	99.5%
TRM formula	0.0803	0.1036	0.1462	0.1619
Monte Carlo	0.0795	0.1027	0.1427	0.1585

5.5 Probabilistic transfer capacity

We observe that our approach is not limited to the determination of transmission reliability margin. Since our approach yields an approximately normal distribution of transfer capability uncertainty U and an estimate (5.7) of the standard deviation of U, this is an alternative way to find the probabilistic transfer capacity as presented in [18, 19, 61, 97, 60]. The probabilistic transfer capacity can be used for system planning, system analysis, contract design and market analysis. Reference [61] suggests promising applications of probabilistic transfer capacity in the new market environment.

5.6 Conclusions

This chapter suggests a way to estimate transmission reliability margin with the formula (5.10). The formula requires estimates of the uncertainty in independent parameters, the evaluation of transfer capability sensitivities, and specification of the degree of safety. The transfer capability sensitivities with respect to many parameters are easy and quick to evaluate once the transfer capability is determined [41, 2]. This ability to quickly obtain sensitivities with respect to many parameters makes it practical to account for the effects of many uncertain parameters in large power system models and improves the central limit theorem approximation used to derive the formula. The formula has been confirmed by comparison with Monte Carlo runs on 8 and 118 bus systems.

The approach includes estimating the statistics of the uncertainty in the transfer capability and thus gives an alternative way to obtain a probabilistic transfer capacity.

The formula provides a defensible and transparent way to estimate transmission reliability margin; in particular, the degree of safety assumed and the sources of uncertainty are apparent in the calculation. The improved estimate of transmission reliability margin will improve the accuracy of available transfer capabilities and could be helpful in resolving the tradeoff between security and maximizing transfer capability. The sensitivities used in the calculation highlight which uncertain parameters are important. Indeed, the calculation provides one way to put a value on reducing parameter uncertainty because a given reduction in uncertainty yields a calculable reduction in transmission reliability margin and this can be related to the profit made in an increased transfer.

Appendix

Let $X_1, X_2, ..., X_m$ be independent, zero mean random variables and write $s_m^2 = \sum_{k=1}^{m} \sigma^2(X_k)$ for the variance of $\sum_{k=1}^{m} X_k$. The approximate normality of $\sum_{k=1}^{m} X_k$ requires a central limit theorem. (Note that the most straightforward version of the central limit theorem does not apply because we do not assume that $X_1, X_2, ..., X_m$ are identically distributed.) A special case of the Lindeberg theorem [10] states that if

$$\lim_{m->\infty} \sum_{k=1}^{m} \frac{1}{s_m^2} \int_{|X_k|>\epsilon s_m} X_k^2 \; dF = 0 \tag{5.11}$$

holds for all positive ϵ then $\frac{1}{s_m} \sum_{k=1}^{m} X_k$ converges in distribution to a normal random variable of mean zero and variance unity.

One useful class of random variables satisfying the Lindeberg condition (5.11) is random variables which are both uniformly bounded and whose variance uniformly exceeds some positive constant. It is also possible to augment the random variables in this class with some normal random variables.

Chapter 6

Uncertainty, probabilistic modeling and optimization

A key engineering judgement in assigning flow limits to transmission lines is determination of an acceptable margin that should be maintained between planned flow levels and the absolute operational limit. Such a margin allows for deviation of the actual line flow from predicted values under the influence of random load and power injection variations.

We start with an illustrative example in which uncertainties in regional temperature variations drive deviations in load patterns from predicted values, and ultimately, deviation in transmission line flows (or so-called "flowgate" flows, constructed as sums of individual line flows). With a set of regional temperature measurements as the fundamental random variable, we step through the necessary modeling assumptions and data requirements to translate temperature statistics (covariances) to covariances in predicted values of line/flowgate flows. These calculations establish the strong dependence on the power transfer distribution factors (PTDFs).

The chapter then continues by establishing a framework for an important optimization problem: given joint probability distributions for variations in bus power injections and loads, maximize nominal power transfers between two buses subject to the constraint that the probability of flowgate flows remaining within limits is above a user specified threshold. Then we describe practical approximation techniques for extending from the case of normally distributed random variables to more general distributions using the Cornish-Fisher expansion.

The chapter finishes by discussing a stochastic optimal power flow formulation which can be addressed with standard power system optimization tools.

6.1 Temperature uncertainty and load response modeling

The motivation for this chapter is the need to treat transmission capability calculations in a manner that accounts for the inherently probabilistic nature of power demand, and, in some market scenarios, of power production. While there are a large number of sources of uncertainty in a power systems operating environment, the work here will focus on continuous variations in demand and production, as opposed to random changes in system structure, as might be associated with generator outages or transmission line tripping.

A key source of uncertainty in power systems operation is the response of power consumers to regional temperature variations. Despite the significant changes in operational procedures that have been brought about under restructuring, one may still assume that operation of the power system is conducted based on advanced planning, using predicted values for load, and that the accuracy of predictions is successively refined as one moves the time horizon of interest closer and closer to real time. A predicted value of load is based on many factors, and load prediction algorithms are a well developed field. However, given an advance load prediction, with a prediction horizon ranging from, say, a week down to an hour, a key source of uncertainty in this prediction will be predicted temperature.

The functional dependence of a load demand level (e.g., a distribution substation) to a set of regional temperature measurements is typically well established in most operating environments. Therefore, we will assume that the following data and models are available:

1. A known set of regional temperature measurement points. On the time horizon of interest, we will assume that the predicted values of temperature are used in generating a load prediction; our probabilistic model therefore focuses only on the deviations in temperature from the predicted values. These deviations will be assumed to be modeled as a vector of jointly gaussian random variables with zero means, denoted $\Delta\mathbf{t}$, with a known covariance matrix $\mathbf{R_t}$.

2. For the time horizon of interest, and based on temperature effects, we will assume that variation in load demands away from predicted values, denoted $\Delta\mathbf{p_D}$, may be expressed as a known linear function of the regional temperature deviations. In particular, we have a known matrix $\mathbf{B}$ such that

$$\Delta\mathbf{p_D} = \mathbf{B}\boldsymbol{\Delta}\mathbf{t}.$$

   Note that $\mathbf{B}$ need not be a square matrix. In some simple cases, one may have temperature measurements at each substation that forms a load point. In this case, $\mathbf{B}$ would be square and strongly diagonal (load at the substation would typically be a strong a function of temperature at that substation, and more weakly dependent on measured temperatures elsewhere). However, in many realistic situations, the locations at which temperatures are measured are not (only) coincident with substation locations. In this case, allowing a general, rectangular matrix to relate temperature variations to load variations is important.

3. The use of the so-called "Power Transfer Distribution Factors," or PTDFs, is described in more detail in section 2.7. Here it suffices to remind the reader that the PTDFs form a matrix describing the linear sensitivity relation between changes in injections or loads (typically restricted to active power

components), and changes in flows on lines. Note that numerically different PTDFs result, depending on whether flow is measured as active power, apparent power, or current; the conceptual construction remains the same for each choice. We will denote the portion of the PTDF matrix that relates our load points of interest to our line flows of interest as $\mathbf{D}$; the line flow deviations themselves will be denoted as $\Delta\mathbf{f_L}$. Therefore, our modeling results in a linear relation between random temperature variation, and the variation in line flow due to this underlying cause. In particular, we have

$$\Delta\mathbf{f_L} = \mathbf{D}\mathbf{p_D} \tag{6.1}$$

and hence

$$\Delta\mathbf{f_L} = \mathbf{D}\mathbf{B}\boldsymbol{\Delta}\mathbf{t}.$$

A key element of this construction is the expectation that the necessary PTDFs for a system will be known and available, and that load modeling and weather predictions for the region of interest ensure that the data contained in the matrices $\mathbf{B}$ and $\mathbf{R}$ may be estimated with reasonable accuracy.

With these elements constructed, standard textbook results in random variables may be brought to bear. In particular, given an underlying driving source of uncertainty that is described as a vector of zero mean, jointly gaussian random variables, any linear function of that vector is itself a vector of zero mean, jointly gaussian random variables. The covariance of the new random vector is directly determined from the original covariance and the matrix forming the linear relation. In particular, the covariance of the line flow variations, denoted as $\mathbf{Q}$, is given by

$$\mathbf{Q} = \mathbf{D}\mathbf{B}\mathbf{R}\mathbf{B}^{\mathbf{T}}\mathbf{D}^{\mathbf{T}}.$$

We will illustrate the interpretation of this covariance matrix in a small example to follow. However, it is useful to note the way in which some qualitative features one might intuitively expect are reflected in the computation. First, even relatively small variances in an underlying random temperature can yield large covariances for line flows. If a group of loads are all dependent on one temperature variation in a like direction, and these loads all contribute in a like direction to a line flow change, the effect of a temperature uncertainty can be greatly magnified. Second, the structure of the computation of $\mathbf{Q}$ makes it highly likely that $\mathbf{Q}$ will have significant off-diagonal components; i.e. variations within certain sets of line flows will often be highly correlated.

6.2 Sample calculation in IEEE 39 bus system

With this background in mind, consider the IEEE 39 bus test system diagram in Figure 6.1. Note that for convenience, each line in this network has been labelled with a number for 1 to 46, as indicated by the underscored numbers adjacent to the reference direction arrow on each line. For the data to follow, we make the simple assumption of the network operating with the generator at bus 30 as slack (i.e., for the incremental load changes examined, the corresponding generation adjustments are made exclusively at this one machine).

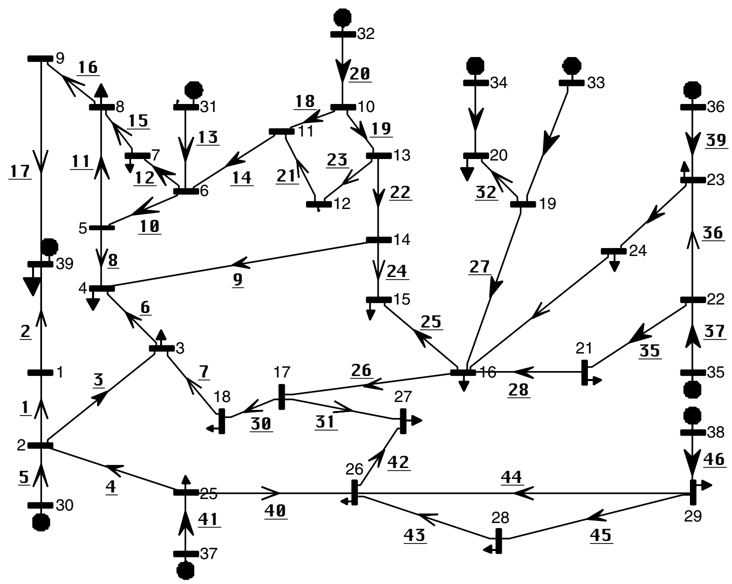

Figure 6.1: IEEE 39 bus test system diagram.

As noted in the preceding section, a key piece of data describing network behavior are the PTDFs for lines and load changes of interest. To illustrate the techniques described here, suppose that the line flows of interest are at lines 2, 3, 4, 6, 7, 14 and 38; all are active power flows, measured at the receiving end of lines, as indicated by reference arrows shown in Figure 6.1. In the notation introduced above, these line flow deviations would be denoted by the variables Δp_{L2}, Δp_{L3}, Δp_{L4} Δp_{L6}, Δp_{L7}, Δp_{L14}, Δp_{L38}. Suppose further that the load variations of interest are active power demands at buses 3, 4, 7, 8, 15, 16, 18, 23, and 24, denoted by the variables Δp_{D3}, Δp_{D4}, Δp_{D7} Δp_{D8}, Δp_{D15}, Δp_{D16}, Δp_{D18}, Δp_{D23}, Δp_{D24}. For the system as considered, with bus 30 as slack bus, the resulting PTDFs are given by the matrix below.

	Δp_{D3}	Δp_{D4}	Δp_{D7}	Δp_{D8}	Δp_{D15}	Δp_{D16}	Δp_{D18}	Δp_{D23}	Δp_{D24}
Δp_{L2}	0.080	0.177	0.261	0.274	0.134	0.114	0.091	0.112	0.114
Δp_{L3}	0.798	0.682	0.609	0.599	0.648	0.639	0.697	0.632	0.640
Δp_{L4}	0.127	0.159	0.150	0.148	0.240	0.263	0.226	0.260	0.263
Δp_{L6}	−0.086	0.604	0.509	0.502	0.273	0.181	0.026	0.178	0.181
Δp_{L7}	−0.115	0.071	0.094	0.091	0.369	0.454	0.667	0.449	0.454
Δp_{L14}	0.019	−0.008	−0.260	−0.247	0.133	0.101	0.052	0.099	0.101
Δp_{L38}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.469	0.077

This matrix is computed from the common format power flow data describing the IEEE 39 bus test system. The authors wish to make clear that the remaining data to follow is purely hypothetical, and is introduced for purposes of illustrating the techniques described in this chapter.

Given that temperature variation (from, perhaps, a day ahead predicted value) is the underlying random element in this analysis, a fundamental piece of data is the covariance matrix for the temperature variation. Such a matrix of covariance values could be estimated from historical time series data comparing (day ahead) predicted temperature to actual observed temperature, under the assumption of a zero mean Gaussian distribution for the error. This type of construction is standard in time series analysis, and will not be treated here.

For illustration, we shall assume that temperature is measured at each of the load buses where demand is assumed to vary: buses 3, 4, 7, 8, 15 16, 18, 23 and 24. It is reasonable to assume that the temperature variations are correlated, so that the covariance matrix will have nonzero off-diagonal elements. However, assuming the buses were ordered so that greater separation in their numeric order represents increasing geographic separation, one would also expect that the magnitude of the off diagonal terms would drop off rapidly away from the diagonal (because wider geographic separation would be associated with lower correlation between temperature errors). For illustrative purposes, below is a representative matrix $\mathbf{R_t}$ of temperature errors in degrees C.

$$\mathbf{R_t} = \begin{matrix} 7.5000 & 0.9375 & 0.2778 & 0.1172 & 0.0600 & 0.0347 & 0.0219 & 0.0146 & 0.0103 \\ 0.9375 & 7.5000 & 0.9375 & 0.2778 & 0.1172 & 0.0600 & 0.0347 & 0.0219 & 0.0146 \\ 0.2778 & 0.9375 & 7.5000 & 0.9375 & 0.2778 & 0.1172 & 0.0600 & 0.0347 & 0.0219 \\ 0.1172 & 0.2778 & 0.9375 & 7.5000 & 0.9375 & 0.2778 & 0.1172 & 0.0600 & 0.0347 \\ 0.0600 & 0.1172 & 0.2778 & 0.9375 & 7.5000 & 0.9375 & 0.2778 & 0.1172 & 0.0600 \\ 0.0347 & 0.0600 & 0.1172 & 0.2778 & 0.9375 & 7.5000 & 0.9375 & 0.2778 & 0.1172 \\ 0.0219 & 0.0347 & 0.0600 & 0.1172 & 0.2778 & 0.9375 & 7.5000 & 0.9375 & 0.2778 \\ 0.0146 & 0.0219 & 0.0347 & 0.0600 & 0.1172 & 0.2778 & 0.9375 & 7.5000 & 0.9375 \\ 0.0103 & 0.0146 & 0.0219 & 0.0347 & 0.0600 & 0.1172 & 0.2778 & 0.9375 & 7.5000 \end{matrix}$$

Next, as noted in Section 6.1, it is necessary to identify a matrix $\mathbf{B}$ which represents the relation of temperature variation to load variation. While it is certainly possible that this quantity could vary with operating conditions, we will assume that a fixed nominal value is established. Given that a given load bus may serve loads that are geographically dispersed about that bus, we will expect that temperature variations throughout the system might have an impact on any given load, but that the strongest impact will be associated with the temperature variation measured at the bus in question. This again results in a matrix with non-trivial off-diagonal elements, but typically diagonally dominant with positive entries. As above, we provide a sample numerical matrix for our example that is meant to be representative of these properties. We assume that $\mathbf{B}$ as given below relates degree C variation in temperature (from its predicted value) with per unit variation in load demand (from its predicted value).

$$\mathbf{B} = \begin{matrix} 0.0120 & 0.0030 & 0.0013 & 0.0008 & 0.0005 & 0.0003 & 0.0002 & 0.0002 & 0.0001 \\ 0.0030 & 0.0120 & 0.0030 & 0.0013 & 0.0008 & 0.0005 & 0.0003 & 0.0002 & 0.0002 \\ 0.0013 & 0.0030 & 0.0120 & 0.0030 & 0.0013 & 0.0008 & 0.0005 & 0.0003 & 0.0002 \\ 0.0008 & 0.0013 & 0.0030 & 0.0120 & 0.0030 & 0.0013 & 0.0008 & 0.0005 & 0.0003 \\ 0.0005 & 0.0008 & 0.0013 & 0.0030 & 0.0120 & 0.0030 & 0.0013 & 0.0008 & 0.0005 \\ 0.0003 & 0.0005 & 0.0008 & 0.0013 & 0.0030 & 0.0120 & 0.0030 & 0.0013 & 0.0008 \\ 0.0002 & 0.0003 & 0.0005 & 0.0008 & 0.0013 & 0.0030 & 0.0120 & 0.0030 & 0.0013 \\ 0.0002 & 0.0002 & 0.0003 & 0.0005 & 0.0008 & 0.0013 & 0.0030 & 0.0120 & 0.0030 \\ 0.0001 & 0.0002 & 0.0002 & 0.0003 & 0.0005 & 0.0008 & 0.0013 & 0.0030 & 0.0120 \end{matrix}$$

As an example of the interpretation of $\mathbf{B}$, suppose one wanted to compute the deviation of loads from predicted values if temperatures at every measurement point deviated from their predicted values by $+1$ degree C. This would be computed simply by multiplying $\mathbf{B}$ times a vector of all positive 1 entries.

As described in our earlier development, we are now prepared to compute the covariance matrix for line flow variations, $\mathbf{Q}$, as

$$\mathbf{Q} = \mathbf{DBRB^TD^T},$$

yielding

$$\mathbf{Q} = \begin{matrix} 0.0011 & 0.0042 & 0.0013 & 0.0020 & 0.0017 & -0.0002 & -0.0002 \\ 0.0042 & 0.0180 & 0.0056 & 0.0075 & 0.0079 & -0.0002 & -0.0012 \\ 0.0013 & 0.0056 & 0.0018 & 0.0022 & 0.0027 & 0.0001 & -0.0004 \\ 0.0020 & 0.0075 & 0.0022 & 0.0038 & 0.0028 & -0.0004 & -0.0004 \\ 0.0017 & 0.0079 & 0.0027 & 0.0028 & 0.0047 & 0.0004 & -0.0008 \\ -0.0002 & -0.0002 & 0.0001 & -0.0004 & 0.0004 & 0.0003 & -0.0001 \\ -0.0002 & -0.0012 & -0.0004 & -0.0004 & -0.0008 & -0.0001 & 0.0003 \end{matrix}$$

This example illustrates how the impact of the network characteristics, as reflected in the PTDFs, may "concentrate" uncertainty on certain line flows. For example, note that the $(2, 2)$ entry in $\mathbf{Q}$ is much larger than any other diagonal entry. The $(2, 2)$ entry in $\mathbf{Q}$ corresponds to the variance of line 3, which is the line connecting buses 2 and 3. While such an interpretation is only a rough heuristic in the case of correlated Gaussian variables, as we have here, we may very roughly interpret the square root of the diagonal covariance entry as a standard deviation. In this rough interpretation, the active power variation on line number 3 is roughly 0.138 per unit. If one computes a corresponding covariance matrix for the bus power deviations, and examines the largest "standard deviation" (square root of a diagonal covariance term), one would find that none exceed 0.038 per unit. Again, one can think of the network layout, and the relative admittances of lines, playing the role of concentrating smaller bus power variations to produce considerably larger line flow variations. This is the key observation the reader should take away from this example.

6.3 Extensions to flowgates and general random injection variation

The preceding development illustrates the procedure for mapping a fundamental random variable (regional temperature variations) through to variations in load demand, and ultimately, to individual line flow variations. In this section, we will broaden our perspective, and consider a random vector of net bus power injections, which may be positive (net generation) or negative (net load).

As we no longer restrict random variation to be associated with deviation from deterministic predicted values, we will drop the prefix of Δ on the new random variables to be introduced below. In particular, we begin with the following

Assumption 1 *There are* n locations *(or* buses*) in the power system. The net real power injection into buses are represented by a column vector* $\mathbf{p}$ *with* n *entries (the units of power flows are often in MW).*

To allow further generality in our formulation, we also wish to extend from flows on individual lines (previously denoted as $\Delta\mathbf{f_L}$), to a more general class of flows composed as weighted sums of individual line flows. Therefore, we proceed with

Assumption 2 *A* flowgate *is a transmission line or a set of transmission lines. Each flowgate has physical limits that must be obeyed. In particular, flows on a constrained flowgate cannot increase in the congested (constraining) direction. There are* m *flowgates in the power system. The column vector* $\mathbf{f} = [\mathbf{f}_1, ..., \mathbf{f}_m]$ *represents the flow in each of the flowgates (associated with each flow is an arbitrarily chosen flow direction).*

Assumption 3 *Flowgate flows obey*[1] *Kirchhoff's laws. The flowgate flows can be expressed as a function of node injections using the following reasonable approximation:*

$$\mathbf{f} = \mathbf{A}\mathbf{p} \tag{6.2}$$

where $\mathbf{A}$ *is a* Power Transfer Distribution Factor (PTDF) *matrix with* m *rows and* n *columns.*

Note that the power transmission distribution factor matrix $\mathbf{A}$ is closely related to the matrix $\mathbf{D}$ introduced earlier in (6.1). In particular, $\mathbf{A}$ has rows formed as weighted sums of rows of $\mathbf{D}$, corresponding to a flowgate flow being defined as weighted sums of individual line flows. The choice to focus on flow gates, rather than individual lines, follows the NERC practice of setting transfer limits for large scale regional transfers in terms of flowgates. With this goal in mind, we continue with

[1] Kirchhoff's laws are a reflection of the physical constraints of the system: the flows on lines are a function of the voltage differences across the lines, and the sums of flows at any node must be precisely zero.

Assumption 4 *Flows on flowgates must be within their physical limits,*

$$\mathbf{b_{low}} <= \mathbf{f} <= \mathbf{b_{up}} \tag{6.3}$$

If a flowgate flow exceeds its operating limit, then the flowgate is said to be congested or constrained[2]*. Since at any given time, the flow in any congested flowgates can either be near the upper limit or near the lower limit, we will without loss of generality (by changing the flow direction) replace (6.3) with the following equation*

$$\mathbf{f} <= \mathbf{b} \tag{6.4}$$

We will find it convenient to describe $\mathbf{A}$ as $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_m]$, where $\mathbf{a}_k$ is the k^{th} row of $\mathbf{A}$ and is of dimension n. Then the flow in flowgate k is given by

$$\mathbf{f}_k = \mathbf{a}_k \mathbf{p}. \tag{6.5}$$

Our main focus in this chapter will be on the following question: what can we say about the uncertainty in flows given uncertainty in the power injections at each node for a given time period? In particular,

1. Given that the node injections have a given probability distribution, what is the probability that $\mathbf{f}_k = \mathbf{a}_k \mathbf{p} <= \mathbf{b}_k$ for flowgate k?
2. Given that the node injections have a given probability distribution, what are the maximum allowable additional power transfers between two buses (or between sets of buses) such that the probability that flowgate flows stay within their flow limits is above a certain threshold?

The optimization problem loosely described above is generally characterized as a "chance constrained optimization" problem in the literature of stochastic programming [11]. The next section introduces some of the technical machinery on probability distributions that we will use in this chapter.

6.4 Background on probability distributions

It is a well-known result that the probability distribution of a random variable can be completely described by its *cumulants* (see, for example, [55]). For most practical problems of interest, the first few cumulants generally are sufficient.

Assumption 5 *We will represent the jth order cumulant for a random variable Z as $\kappa_j(Z)$. When there is no ambiguity, we will suppress the functional dependence.*

[2] Generally, when a flowgate flow exceeds its limit, the power injections at different buses will be "re-dispatched" to bring them back to within their limits. This re-dispatch is usually accomplished most economically by ramping up cheaper generators at one end of the congested flowgate, and ramping down more expensive generators at the other end of the congested flowgate. In some cases, transmission overloads are resolved by removing the transmission line. In some extreme cases, re-dispatch of power injections are done by curtailing loads in congested pockets of the system.

We note that for the normal distribution, only the mean and variance are generally non-zero; all higher order cumulants are zero. Therefore higher-order cumulants capture departures from normality for non-normal distributions. For most practical distributions, most of the departure is captured in κ_3 and κ_4. Therefore, in this chapter all cumulants of order higher than 4 will be assumed to be zero, i.e, $\kappa_j = 0$, for $j > 4$. We note the results in this chapter are general and are independent of this approximation.

For convenience, we define the first few cumulants for a random variable Z (here the operator $E[Z]$ describes the expected value of Z):

$$\kappa_1 = E[Z] \text{ (also mean)} \tag{6.6}$$

$$\kappa_2 = E[Z - \kappa_1]^2 \text{ (variance or second central moment)} \tag{6.7}$$

$$\kappa_3 = E[Z - \kappa_1]^3 \text{ (third central moment)} \tag{6.8}$$

$$\kappa_4 = E[Z - \kappa_1]^4 - 3\kappa_2^2 \tag{6.9}$$

Cumulants have the very desirable property that they are additive for linear sums of independent random variables. More precisely, for a random variable $Z = \Sigma_i a_i X_i$, where the X_i's are independent random variables and the a_i's are constants,

$$\kappa_j(Z) = \sum_i a_i^j \kappa_j(X_i) \tag{6.10}$$

When the X_i's in (6.10) are not independent, then the cumulants will include "cross-cumulant" terms. In this chapter, we will only worry about the cross-cumulants for the first two cumulants, and make the approximation that higher-order cross-cumulant terms are zero. There are two reasons why this is a practical approximation.

1. For normal distributions, the approximation is exact, i.e., the cross-cumulant terms are zero; indeed all cumulant terms of order 3 and above are zero. Therefore, the approximation is valid for probability distributions that are only "slight" departures from the normal distribution.

2. In general, it becomes increasingly less accurate to estimate cross-cumulant terms for higher-order cumulants. Therefore even if we do include the higher-order cross-cumulants, we will have to live with the inherent inaccuracy of such data.

Therefore, we will use the following generalization to (6.10), when the X_i's are not independent random variables.

$$\kappa_1(Z) = \sum_i a_i \kappa_1(X_i) \tag{6.11}$$

$$\kappa_2(Z) = \sum_i \sum_l a_i a_l \kappa_2(X_i, X_l) \tag{6.12}$$

$$\kappa_j(Z) = \sum_i a_i^j \kappa_j(X_i) \quad , j = 3, 4, 5, ... \tag{6.13}$$

where $\kappa_2(X_i, X_l)$ is the (i, l) entry of the covariance matrix Σ for the X_i's. Note that (6.13) is an approximation. Writing

$$\mathbf{a} = [a_1, a_2, ...] \tag{6.14}$$

$$\kappa_i(\mathbf{X}) = [\kappa_i(X_1), \kappa_i(X_2), ...]^t \tag{6.15}$$

$$\mathbf{a}^j = [a_1^j, a_2^j, ...] \tag{6.16}$$

we can write (6.13) in more compact vector form as

$$\kappa_1(Z) = \mathbf{a}\kappa_i(\mathbf{X}) \tag{6.17}$$

$$\kappa_2(Z) = \mathbf{a}\Sigma\mathbf{a}^t \tag{6.18}$$

$$\kappa_j(Z) = \mathbf{a}^j\kappa_j(\mathbf{X}) \qquad , j = 3, 4, 5, ... \tag{6.19}$$

(Note that the treatment for the higher order cumulants is different from the treatment of the cumulant of order 2 because the cross-cumulant terms are neglected.)

In the sequel, we will find it useful to "standardize" a random variable Z by using the following transformation

$$\widetilde{Z} = \frac{Z - E[Z]}{\sigma_Z} \tag{6.20}$$

where σ_Z is the standard deviation of Z.

For a standardized random variable, the cumulants can be related to the more familiar statistical terms

$$\kappa_1 = 0 \tag{6.21}$$

$$\kappa_2 = 1 \tag{6.22}$$

$$\kappa_3 = \text{ skewness} \tag{6.23}$$

$$\kappa_4 = \text{ kurtosis} \tag{6.24}$$

We end this section with an extremely important theorem in statistics called the Cornish-Fisher expansion [26] (see also [54] for more detail and other references). The power in the theorem lies in its ability to relate random variables with arbitrary probability distributions to the normal distribution.

Proposition 1 (Cornish-Fisher) *Let Z be a random variable standardized according to (6.20) with probability distribution $g(z)$. Let $N(x)$ be the probability distribution function of a standardized normal random variable, i.e., a normal distribution with mean* 0*, and variance* 1*. Then*

$$\int_{-\infty}^{y} g(z)dz = \int_{-\infty}^{k} N(x)dx \quad \textit{is equivalent to} \quad k = f(y) \tag{6.25}$$

where

$$f(y) = y - \frac{1}{6}(y^2 - 1)\kappa_3(Z) - \frac{1}{24}(y^3 - 3y)\kappa_4(Z) + \frac{1}{36}(4y^3 - 7y)\kappa_3(Z)^2 + \textit{higher order cumulant terms} \tag{6.26}$$

(See [54] for the inverse relationship, i.e., y as a function of k.)

We stress that the Cornish-Fisher expansion stated in (6.26) is for standardized random variables. So $g(z)$ must represent the probability distribution function of the standardized random variable Z. Also, it is very important to verify that the higher order terms are indeed small before they can be neglected. For example, if the skewness is much larger than 1, then one should include higher order skewness terms because they cannot be discarded. See [54] for more details.

Let $\Psi(x)$ represent the cumulative probability distribution of a standardized normal probability distribution. There are standard tables for $\Psi(x)$. We now solve the following two practical cases of interest.

1. If y is given, what is $\alpha = \int_{-\infty}^{y} g(z)dz$?
2. If α is given, and we need the condition that $\alpha <= \mathbf{prob}\{Z <= y\} = \int_{-\infty}^{y} g(z)dz$, what condition should the upper limit y satisfy?

The first case can be solved from (6.25) as

$$\alpha = \Psi(f(y)) \tag{6.27}$$

For the second case, we proceed as follows. The equation

$$\alpha <= \int_{-\infty}^{k} N(x)dx = \int_{-\infty}^{y} g(z)dz \tag{6.28}$$

is equivalent to

$$\Psi^{-1}(\alpha) <= k = f(y) \tag{6.29}$$

The first equality follows from the Cornish-Fisher relation (6.26). The second inequality holds because $\Psi(x)$, which is a cumulative probability distribution function, is a monotonically increasing function of x. Therefore

$$\alpha <= \mathbf{prob}\{Z <= y\} \text{ is equivalent to } \quad \Psi^{-1}(\alpha) <= f(y) \tag{6.30}$$

To summarize,

1. We will represent the probability distribution function for any random variable by its first four cumulants only. We note however, that in general, the methods in the chapter are valid regardless of how many cumulants are considered. Neglecting higher order cumulants is convenient (but not essential) in presenting the basic ideas of this chapter. Moreover, consideration of up to four cumulants is a very practical and reasonable approximation for many probability distributions.

2. The only cross-cumulant terms considered when two or more random variables are involved will be the covariance matrix term. Again, this approximation is made for the sake of convenience only; the general results in this chapter are valid regardless of this approximation.

3. We can use the Cornish-Fisher expansion to express the probability of an event that an arbitrary random variable lies in a certain interval in terms of a standardized normal distribution.

6.5 Probability of transmission congestion in flowgates

In this section, we will estimate the probability of transmission congestion for a given time period, given exogenous uncertain inputs. In particular, assume that we are given the cumulants for $\mathbf{p}$ (and the second-order cumulant term, which is the covariance matrix Σ). Then what is the probability that, for a flowgate k, $\mathbf{f}_k <= \mathbf{b}_k$? Or, from (6.5),

$$\text{What is the probability that } \mathbf{a}_k\mathbf{p} <= \mathbf{b}_k? \tag{6.31}$$

From (6.31), we get

$$\mathbf{prob}\{\mathbf{f}_k <= \mathbf{b}_k\} = \mathbf{prob}\{(\mathbf{f}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k} <= (\mathbf{b}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k}\} \tag{6.32}$$

provided the standard deviation of $\mathbf{f}_k$, $\sigma_{\mathbf{f}_k}$ is non-zero (if $\sigma_{\mathbf{f}_k} = 0$, then (6.31) becomes a trivial deterministic problem).

Let $Z = (\mathbf{f}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k}$, and note that Z is now in standardized form. From (6.19), we get the following cumulants for Z.

$$\kappa_1(Z) = 0 \tag{6.33}$$

$$\kappa_2(Z) = 1 \tag{6.34}$$

$$\kappa_j(Z) = \frac{\mathbf{a}_k^j \kappa_j(\mathbf{p})}{\sigma_{\mathbf{f}_k}} \quad , j = 3, 4, 5, ... \tag{6.35}$$

Now, from (6.13),

$$E[\mathbf{f}_k] = \kappa_1(\mathbf{f}_k) = \mathbf{a}\kappa_1(\mathbf{p}) \tag{6.36}$$

$$\sigma_{\mathbf{f}_k}^2 = \kappa_2(\mathbf{f}_k) = \mathbf{a}_k \Sigma \mathbf{a}_k^t \tag{6.37}$$

where Σ is the covariance matrix for $\mathbf{p}$.

Rewriting (6.32) in terms of Z gives

$$\mathbf{prob}\{\mathbf{f}_k <= \mathbf{b}_k\} = \mathbf{prob}\{Z <= (\mathbf{b}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k}\} \tag{6.38}$$

Using the result (6.27) from the previous section, we get

$$\mathbf{prob}\{\mathbf{f}_k <= \mathbf{b}_k\} = \mathbf{prob}\{Z <= (\mathbf{b}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k}\} = \Psi\left(f\left((\mathbf{b}_k - E[\mathbf{f}_k])/\sigma_{\mathbf{f}_k}\right)\right) \tag{6.39}$$

where $E[\mathbf{f}_k]$ and $\sigma_{\mathbf{f}_k}$ are given by (6.37), and f is the functional form of the Cornish-Fisher expansion (6.26). Note that the cumulative normal distribution function Ψ is available as standard tables.

We note that it is difficult to generalize the result to the vector case (to find out $\mathbf{prob}\{\mathbf{Ap} <= \mathbf{b}\}$), because the Cornish-Fisher expansion is not applicable to vectors. Indeed, even for the relatively simple case when $\mathbf{p}$ has a normal distribution (and each element of $\mathbf{p}$ is independent), it is surprisingly difficult to obtain general analytical results for $\mathbf{prob}\{\mathbf{Ap} <= \mathbf{b}\}$ (except for special cases). Indeed, even numerical results for such integrals are hard for large dimensions. For example, see [35, 36]; Genz's website also contains a wide variety of research papers and useful software.

6.6 Numerical Example

Consider the 6 bus example introduced in section 1.4.5. We will now demonstrate how the Cornish-Fisher expansion can be used to estimate the probability that the flow on the line connecting busses 2 and 5 will exceed its rating of 100 MW. We make the following simplifying assumptions:

1. The load at each bus is an independent random variable. Bus 1 is assumed to be the slack bus that supplies all the load.
2. The mean load at each of the 6 buses is 900 MW.
3. The standard deviation of the load at each bus is 90 MW.
4. We will consider 2 separate cases; one will be a purely normal distribution, and the other will be a non-normal distribution.
   (a) The non-normal distribution is specified completely by each bus load's skewness and kurtosis (in addition to the mean and standard deviation). We assume that each bus load's skewness is 0.9, and kurtosis is 0.1.
   (b) For the normal distribution, all cumulants ordered higher than 2 are zero.

The appropriate power transfer distribution factor ($\mathbf{a}_k$) for the line connecting bus 2 and 5 is given by

$$\mathbf{a}_k = [0,\ 0.0993,\ -0.0342,\ 0.0292,\ -0.1927,\ -0.0266] \tag{6.40}$$

It is important to note that we need to measure the probability of congestion in *both* directions of this transmission line.

Using the formulas in (6.39), (6.26), we get the following results.

1. For the normal distribution, the probability of congestion in one direction is zero, and the probability of congestion in the reverse direction is 73%.
2. For the normal distribution, the probability of congestion in one direction is zero, and the probability of congestion in the reverse direction is 71%.

6.7 Maximizing probabilistic power transfers

We next tackle a more complicated problem. Suppose that probability distributions for certain node injections are known for a prospective time period. For example, since loads at buses are related to temperature, node loads would be known with a certain probability distribution. Suppose that we would like to maximize power transfers between a subset of other buses subject to the constraint that no flowgate flow should exceed its limit with a given probability α. How should we approach this problem?

To answer this question, we make the following assumptions regarding the probability distribution of node injections.

Assumption 6 *Let* $\mathbf{x_r}$ *of dimension* n_r *represent those buses whose net power flows are assumed to have a known probability distribution. Without loss of generality, we will label the first* n_r *buses to be these buses. Then we can write* $\mathbf{p} = [\mathbf{x_r}^t \ \mathbf{x_d}^t]^t$ *where* $\mathbf{x_d}$ *is a vector of dimension* $n_d = n - n_r$. *Similarly partition the PTDF matrix* $\mathbf{A}$ *as* $\mathbf{A} = [\mathbf{A_r} \ \mathbf{A_d}]$. *The vector* $\mathbf{x_d}$ *represent a vector of parameters.*

We want to solve the problem of the type below.

$$\text{Maximize } \mathbf{cx_d} \tag{6.41}$$

subject to the constraints that

$$\alpha <= \mathbf{prob}\{\mathbf{Ap} <= \mathbf{b}\} \tag{6.42}$$

$$\text{or, equivalently,} \quad \alpha <= \mathbf{prob}\{\mathbf{A_r x_r} + \mathbf{A_d x_d} <= \mathbf{b}\} \tag{6.43}$$

Writing the k^{th} rows of $\mathbf{A_r}$, $\mathbf{A_d}$ as $\mathbf{A_{r}}_k$, $\mathbf{A_{d}}_k$ respectively, (6.43) can be rewritten as

$$\begin{aligned}
&\alpha <= \mathbf{prob}\{\mathbf{A_r}_k\mathbf{x_r} + \mathbf{A_d}_k\mathbf{x_d} <= \mathbf{b}_k\} \\
&\alpha <= \mathbf{prob}\{\mathbf{A_r}_k\mathbf{x_r} <= \mathbf{b}_k - \mathbf{A_d}_k\mathbf{x_d}\} \\
&\alpha <= \mathbf{prob}\{(\mathbf{A_r}_k\mathbf{x_r} - E[\mathbf{A_r}_k\mathbf{x_r}])/\sigma_{\mathbf{A_r}_k\mathbf{x_r}} <= (\mathbf{b}_k - \mathbf{A_d}_k\mathbf{x_d} - E[\mathbf{A_r}_k\mathbf{x_r}])/\sigma_{\mathbf{A_r}_k\mathbf{x_r}}\} \\
&\text{for} \quad k = 1, ..., m
\end{aligned} \tag{6.44}$$

For notational convenience, define

$$m_k = E[\mathbf{A_r}_k\mathbf{x_r}], \ \sigma_k = \sigma_{\mathbf{A_r}_k\mathbf{x_r}} \tag{6.45}$$

and

$$Z_k = \frac{\mathbf{A_r}_k\mathbf{x_r} - E[\mathbf{A_r}_k\mathbf{x_r}]}{\sigma_{\mathbf{A_r}_k\mathbf{x_r}}} = \frac{\mathbf{A_r}_k\mathbf{x_r} - m_k}{\sigma_k} \tag{6.46}$$

Note that Z_k is now a standardized random variable, i.e., it has mean 0 and variance 1.

Moreover, from (6.13), we can write

$$m_k = \mathbf{A_r}_k\kappa_1(\mathbf{x_r}), \ \sigma_k^2 = \mathbf{A_r}_k\Sigma\mathbf{A_r}_k^t \tag{6.47}$$

where Σ represents the covariance matrix of $\mathbf{x_r}$. Then (6.44) can be written as

$$\alpha <= \mathbf{prob}\{Z_k <= (\mathbf{b}_k - \mathbf{A_d}_k\mathbf{x_d} - m_k)/\sigma_k\} \quad , k = 1, ..., m \tag{6.48}$$

Since Z_k is a standardized random variable, we can now combine (6.30) and (6.48) to get

$$\Psi^{-1}(\alpha) <= f\left((\mathbf{b}_k - \mathbf{A_d}_k\mathbf{x_d} - m_k)/\sigma_k\right) \quad , k = 1, ..., m \tag{6.49}$$

where f is the functional form of the Cornish-Fisher expansion given by (6.26).

Now the optimization problem becomes

$$\text{Maximize } \mathbf{cx_d} \tag{6.50}$$

subject to

$$\Psi^{-1}(\alpha) <= f\left((\mathbf{b}_k - \mathbf{A}_{\mathbf{d}k}\mathbf{x}_\mathbf{d} - m_k)/\sigma_k\right) \quad , k = 1, ..., m \tag{6.51}$$

which is a deterministic optimization problem in standard form and can be solved by standard means. We note that when $\mathbf{x_r}$ are jointly normal distribution, the Cornish-Fisher transformation function f in (6.26) becomes the identity map, since all cumulants above order 2 are zero for a normal distribution. Then the above optimization problem becomes a standard linear programming problem in $\mathbf{x_d}$.

6.8 Numerical Example

This section uses the results of section 6.7 to estimate the maximum transfer capacity between 2 busses for the 6 bus example introduced in section 1.4.5. We make the following simplifying assumptions:

1. The load at each bus (except bus 2) is an independent random variable with a normal distribution. Bus 1 is assumed to be the slack bus that supplies all the load.
2. The mean load at each bus (except bus 2) is 200 MW.
3. Standard deviation of the load at each bus (except bus 2) is 44.72 MW.

α	max transfer (MW)
0.99999	-39.67
0.9999	-15.26
0.999	12.85
0.99	47.02
0.9	93.74
0.8	113.41
0.5	151.06
0.25	181.22
0.1	208.37
0.05	224.62
0.01	255.09
0.001	289.26
0.0001	317.38
0.00001	341.79
0	infinity

Table 6.1: Maximum transfers from bus 1 to bus 2 as a function of no-congestion probability α.

Given these assumptions, we find the maximum transfers from bus 1 to bus 2 so that the probability of no transmission line overload for each transmission line (in *both directions*) is above a threshold probability α. Thus α is the probability of no congestion for that maximum transfer. This involves solving the linear programming problem (6.51). The solution is described in Table 6.1.

Table 6.1 demonstrates that the maximum transfer capacity varies highly non-linearly with the threshold probability α. For very low threshold probabilities, the maximum transfer capacity is very high; for high threshold probabilities, the maximum transfer capacity is more constrained.

6.9 Stochastic optimal power flow

In determining the optimal capability of a transmission network, and more specifically, limits on its ability to transfer power along a transmission corridor, it is natural to seek an adaptation of existing power system optimization tools to this problem. A wide class of optimization problems relating to real time and near real time operational decisions in the grid are grouped under the general framework of Optimal Power Flow (OPF). However, new objectives in a competitive environment [46], [53], the associated focus on congestion management to facilitate competition [78], and the increasing volatility of dispatch and operating conditions raise significant new challenges in OPF [65]. Adding to these challenges is the need to explicitly treat uncertainty in optimization problems relating to transmission network limits. While some efforts in this direction exist in the literature (see, for example, [79], [31], and [32] for early work), the greater levels of uncertainty introduced by market based dispatch make the need for this class of methods much more pressing.

In a competitive power systems environment, the policy decision to manage the provision of power via day ahead or hour-by-hour auctions inevitably introduces stochastic behavior in the amount of power produced in any period, as the supply versus demand equilibrium will be dictated by market variations and by time varying producer and consumer preferences. Averaged over the large number of producers and consumers, this variation may be approximated as a continuous quantity. However, these uncertainties in amount of power production are further exacerbated by uncertainties in the geographic point of connection. Given a portfolio of individual generating plants that may bid into the market to meet demand, their various geographic locations correspond to different points of connection to the electrical circuit that comprises the power system. Given the limits on the power or current carrying capability of transmission lines, cables, and transformers, and given the stochastic nature of production and load, the challenge in setting limits on transmission capability so as to maintain an acceptably small probability of line overload is clear. Yet in contrast to this clear need, present day industry standards for security, through the "$n-1$ criterion," are purely deterministic. Satisfaction of the $n-1$ security criterion simply provides a guarantee of worst case performance in the face of those operating conditions chosen to be included in the contingency list. Therefore, consideration of relative probability of conditions which might cause overload is entirely "buried" in the choice of the contingency list to be studied. If a threatening condition is included in the contingency list, the system limits (such as ATC) are set such that this condition is survivable with probability one. If a threatening condition is judged to be of low enough probability that it is not included in the contingency list, ATC limits based on $n-1$ criterion do not reflect the threat in any way. Likewise, threats associated with multiple events are not reflected at all. Clearly, given the growing volatility of operating conditions observed in competitive markets, this does not appear to be a viable means of setting secure ATC limits in the future electric power grid. The shortcomings of the $n-1$ security criteria for a competitive environment are clearly described in [86].

We begin from the simple representative example of a deterministic formulation of OPF (2.1) in section 2.6 and we indicate how uncertainty is included in the problem formulation. With the notation developed in section 2.6, we can state a representative form of stochastic optimal power flow problem designed to identify the maximum power that may be shipped along a transmission corridor. Recognizing that stochastic variation in production and demand will makes these current flows random quantities, it is the expected value of flow that is to be maximized.

Inequality constraints to be satisfied in the formulation below include limits on the magnitude of current flow in each line of the network, upper and lower limits on bus voltage magnitude at each bus, and upper and lower limits on active and reactive power at each bus. All these inequality constraints are formulated probabilistically as chance constraints. That is, recognizing that in a stochastic setting there is always small but finite probability that there will be no operable solution for which all constraints are satisfied, these inequalities are required to be satisfied with some high probability. For simplicity below, all are required to meet a uniform probability of satisfaction of $1-\epsilon$. ϵ is interpreted as the probability that any operating constraint would be exceeded.

As a final observation before introducing our representative formulation, it is also important to note the class of OPF operating controls *not* being addressed; namely those relating to discrete decisions, which would transform the problem to a mixed integer/nonlinear program. Treatment of issues of discrete decisions presents significant challenges in deterministic OPF [58]. While optimization over discrete decisions in a stochastic environment has been treated recently in the context of traditional unit commitment [84], [85], these works do not introduce the challenging element of detailed network power flow constraints. It would appear that treatment of full power flow constraints, along with mixed discrete/continuous decision variables in a stochastic framework, challenges the current state of the art available in optimization algorithms and software.

With this background, our representative stochastic optimization formulation of the ATC calculation may be summarized as shown below.

$$\max_{\mathbf{V}} E[\sum_{k \ in \ \Gamma} \mathbf{i_k}]$$

subject to

$$Prob[\mathbf{I}^{min} < \mathbf{i}(\mathbf{V}) < \mathbf{I}^{max}] > 1-\epsilon$$

$$Prob[\mathbf{V}^{min} < |\mathbf{V}| < \mathbf{V}^{max}] > 1-\epsilon$$

$$Prob[\mathbf{P}^{min} < Re\{\mathbf{S}(\mathbf{V})\} < \mathbf{P}^{max}] > 1-\epsilon$$

$$Prob[\mathbf{Q}^{min} < Im\{\mathbf{S}(\mathbf{V})\} < \mathbf{Q}_i^{max}] > 1-\epsilon$$

where the complex current flow on branches, $\mathbf{i}$, satisfies

$$\mathbf{i}(\mathbf{V}) = \mathbf{y}. * [\mathbf{A}^T\mathbf{V}]$$

and the complex power absorbed into the network from each node, $\mathbf{S}$, is given by

$$\mathbf{S}(\mathbf{V}) = \mathbf{V}. * \mathbf{A}(conj(\mathbf{y}. * [\mathbf{A}^T\mathbf{V}])) + \sigma$$

where $\mathbf{A}$ is the node-to-branch incidence matrix, σ is a complex-valued random vector representing uncertainty in active and reactive power injection/demand at each bus, and the limit thresholds $\mathbf{I}^{min}$, $\mathbf{I}^{max}$, $\mathbf{V}^{min}$, $\mathbf{V}^{max}$, $\mathbf{P}^{min}$, $\mathbf{P}^{max}$, $\mathbf{Q}^{min}$ and $\mathbf{Q}^{max}$ are all given real-valued parameters, as described in section 2.6.

Given the simple quadratic form of the constraint equations, with random variations entering linearly, the linearized version of this problem directly reduces to the standard form of a linear program with chance constraints. In particular, letting $\mathbf{z}$ represent the real and imaginary parts of the bus voltage magnitudes, linearization of the optimization above about a given operating point produces a problem with the structure

$$\max_{\mathbf{z}} E[\mathbf{c}^T \mathbf{z}]$$

subject to

$$Prob[\mathbf{A_1} z + \mathbf{A_2} \sigma < \mathbf{b}] > 1 - \epsilon$$

Note the great simplification inherent in the fact that the random variables σ representing uncertainty in injection/demand enter linearly; the effect of the linearized power injection equations is simply one of shifting the mean of random variables formed as linear transformation on σ. In the case for which σ is a vector of jointly Gaussian random variables, the requirement that the inequality constraints be satisfied with given probability transforms directly to a set of deterministic constraints. A specific numeric example of this class of computation is provided in section 6.3.

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Interactions among Limits during Maximum Loadability and Transfer Capability Determination

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Abstract: The main idea of this paper is to illustrate the interaction between various kinds of power system operational limits and corrective actions. In particular, the paper focuses on the interplay between line flow limits and active generation redispatch as the corrective action. The proposed methodology handles a wide range of flow limits by explicit inclusion of line flow equations into the set of the system model equations. Four options for active generation redispatch are discussed. The applicability of the methodology in deregulated environments for operating a power system within its security constraints by Security Coordinators or Transmission Providers is discussed. The power system model is based on a computationally efficient point of collapse power flow for tracing steady state behavior due to slow system parameter variations. Results of tests obtained with help of the IEEE-39 and IEEE-118 bus systems are given to illustrate the performance of the proposed methodology.
Keywords: Transfer capability, Redispatch, Maximum loadability, Power system security, Cascading overloads.

1 Introduction

Power system security is defined as the ability of the system to meet load under a credible set of contingencies. Insecurity is the inability to meet all or part of the load. Insecurity can be due to [1]:

- Transient stability limits or voltage collapse limits.
- The total available generation smaller than the demand.
- Component (line and/or transformer) overloads.

Transient stability considerations provide a realistic limit to the amount of power that can be delivered to a region of the system. Such a kind of analysis is based on an algebraic-differential set of equations, and will not be studied. Unlike transient instability, voltage collapse considerations can be based on power flow model, if some assumptions are made. If total generation is smaller than demand, the system frequency drops to unacceptable values. This may result in immediate under-frequency load shedding. In an interconnected system this condition also result in unscheduled interchange flows from other system areas. Component overloads can result in the component outage, which translate into either an immediate load outage (if the component is radial), or a subsequent stability limit or voltage collapse limit leading to an outage. It can also lead to subsequent cascading overloads.

2 Interactions among Limits

Security limits can be divided into two groups:

- Hard security limits (HL). These are limits that cannot be overcome by any of the acceptable corrective actions.
- Soft security limits (SL). We define these as limits that, once reached, can be tolerated or regulated by some action.

Flow limits (FL) are generally soft and voltage collapse (VC) limits are often considered as HL. A generation limit, as defined in previous section, is a HL. However, reaching a limit at any particular generator is a SL and can be easily handled. A true HL is one in which no (acceptable) corrective action is available (NCAA). Namely, when dealing with security limits one of the most important issues is determination of the control actions to alleviate or avoid the problem. Security problems can be relieved by: load reduction, generation redispatch, using phase shifters, and ensuring additional reactive support to the system.

From power system operation point of view a few logical questions arise if a soft security limit is reached:

- Is it possible to handle the limit and continue to operate the system within its limits?
- Is the chosen corrective action successful?
- When the corrective action takes place, how "far" can the system operate before reaching the next limit (soft or hard)?
- What kind of next limit the system is likely to encounter?
- How many soft limits can be handled before reaching a hard system limit?
- How does soft limits handling affect the ultimate hard limit?
- Complexity, practicality?
- What is the cost of extending the loadability margin?

Some limit interactions have been addressed in literature [1,2,3], among themselves as well as with various kinds of corrective actions. It is well known that reactive power generation limits influence the VC limit [2,3]. Excessive active power generation limits lead to a NCAA limit. Using phase shifters for control purposes is a subject of ongoing research. Ensuring additional reactive support to the system affect VC and NCAA limits. Security problems can always be relieved by load reduction. This kind of control action affects all system limits. Voluntary curtailment of load by customers upon notice relieves slow-developing problems that result from changing system conditions. Involuntary load interruption falls in the category of load shedding, and is limited to immanent and severe conditions. The methodology proposed here consists of a multi-purpose powerful tool capable of answering these questions through simulation of different system scenarios. The concept is illustrated in Figure 1. The main idea is to handle flow limits by means of generation redispatch as the corrective action, before reaching system hard limits.

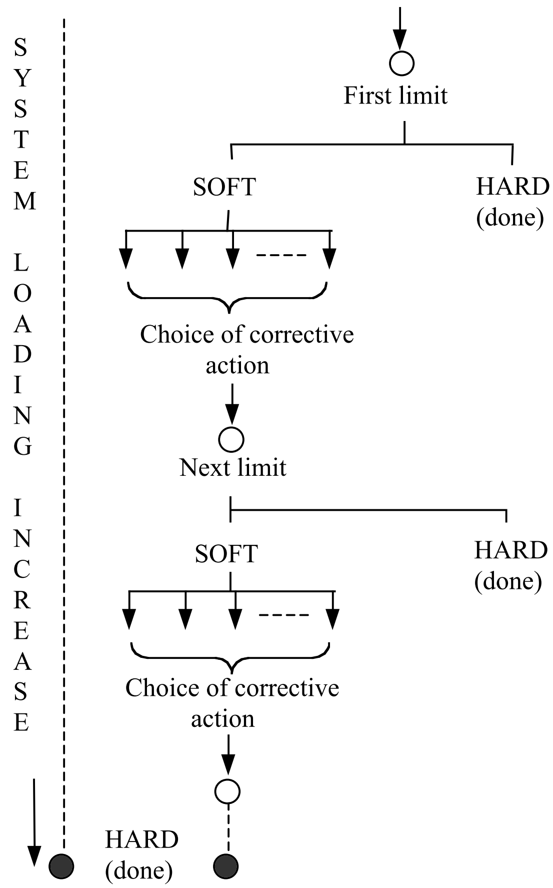

Figure 1 Concept of the methodology

3 Determination of Limits

One of the most powerful tools employed for voltage collapse analysis is the continuation power flow [2,3,4,5]. This methodology enables one to determine the system load margin [5,6], the critical bus [7,8], and control actions to avoid the problem [5,9]. The application of a continuation power flow for voltage collapse analysis ranges from the bifurcation point determination [5,6,9] to contingency screening, when the critical lines most likely to drive the system to voltage collapse are determined [7]. The computational time associated with a continuation power flow may be a barrier, but the accurate results obtained render this methodology a benchmark for other methodologies. The problem studied in this paper gives to a continuation power flow an additional special feature: handling the line flow limits. In this paper the system model is based on an efficient variant of continuation power flow termed as "maximum loadability" or Point of Collapse Power Flow, a sparse vectorized, Matlab implementation of a Newton power flow. Among many, the following capabilities are provided [10]:

- Reading of a Common Format and PTI format file into "Data Dictionary" structures (a slight "raw" variant of the Power System Application Data Dictionary is used).
- Solution of an ordinary power flow with area interchange controls.
- Solution of a "maximum loadability" power flow without area interchange controls.
- Solution of a "maximum loadability" power flow with area interchange controls.
- Rapid and efficient computation of Power Transfer Distribution Factors (PTDF) and Outage Transfer Distribution Factors (OTDF).
- Graphic display of all the results.

At the heart of the model is an extremely efficient method for the construction of the Jacobian matrix and complete vectorization of all operations []. One definition of voltage collapse and maximum loadability is when the Jacobian matrix becomes singular. The method by [4] directly computed this point by appending an explicit Jacobian singularity condition to the continuation problem. Such an approach has excellent convergence characteristics, but only in the immediate vicinity of the nose point. An alternative that is often used in continuation methods is to swap variables. As the Jacobian becomes near singular, the loadability parameter λ ceases to be an independent variable. The continuation variable becomes something else, usually, voltage. Furthermore, there is never a need to solve a set of equations larger than the optimal set. Of course, reaching the nose point is often impractical, unless voltage collapse occurs at a high voltage value. In most systems, there are many practical and operational reasons why a simple constraint on the voltage magnitude is a more significant and limiting constraint.

4 Redispatch as the Corrective Action

The Operational Limit Boundary (OLB) is a surface in bus demand space (demand space variables are the variables associated with load powers, both active and reactive) that corresponds to one of the three operational limits mentioned above. Much work has been done in a load space to control load direction to avoid the system limits [4,8,9]. The load pattern is relatively uncontrollable due to the uncontrollable consumer demand. Little work has been done in the generation space (active and reactive generation levels) to control generation pattern to avoid the system limits. The generation pattern is easier to control than the load pattern. The method proposed in [11] deals with generation space and the gradient search method to find the generation direction that maximizes the total generation on the OLB. Active and reactive generation, load powers and system basic state variables are related to each other by the power flow equations

$$f(P_g, P_d, x) = 0 \qquad (1)$$

The equations that determine the system behavior depend on the assumed generation dispatch policy and the demand behavior. From the perspective of system demands, the simplest option is to consider that the demands are independent of system basic variables. From the perspective of generation, the simplest assumption is to consider that all generators are adjusted in a predetermined direction (Fixed Dispatch Policy). This direction can be entirely arbitrary, or, more likely, chosen in a rational manner. In Fixed Dispatch Policy generation is adjusted starting from some known generation vector P_g^0 until the power flow equations are satisfied for any given level of demand

$$P_g = P_g^0 + k_g \Delta P_g \qquad (2)$$

where k_g is a generation level parameter, and ΔP_g is a predetermined generation increase direction vector. Demand can increase only until a boundary limit is met. For generation limits it is reasonable to assume that generation limits, when reached, alter the direction ΔP_g. When generator j reaches an upper limit, the $j-th$ component of ΔP_g becomes zero to prevent further generation increases from exceeding the limit. Only when all components of ΔP_g become zero has the system reached an OLB based on insufficient generation. Why are all these mentioned here? Because of the simple reason: the core idea of line flow limits handling, by means of active power generation redispatch, is altering the direction ΔP_g when a line flow limit is reached, and steering the system further until the next flow limit, insufficient generation or voltage collapse limit is reached. To perform this, the following steps are proposed:

- With help of the point of collapse power flow, trace the system PV curve as a function of the defined load increase direction.
- If a flow limit is reached, choose a generator pair most likely to act. Take control action to handle the limit with the help of the generator pair.
- Keep tracing the PV curve until another flow limit, generation, or voltage collapse limit is reached.

5 The problem of flow limit equation inclusion

Handling line flow limits by active power generation redispatch gives to the point of collapse power flow an additional special feature. Flow limit equations are considered explicitly in the set of the equations. This is reflected by the inclusion of a new row and a new column to the system Jacobian matrix. The augmented Jacobian, in Newton solution, has the form:

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ rhs1 \\ rhs2 \end{bmatrix} = \begin{bmatrix} H & N & k1 & \\ M & L & & k2 \\ FF1 & & kF1 & \\ & FF2 & & 0 \end{bmatrix} \times \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta\lambda \\ TS \end{bmatrix} \qquad (3)$$

Vector $FF2$ contains the partial derivatives of the flow limit equation with respect to the system state variables. Therefore, it is a vector with no more than four nonzero elements. Vector $k2$ is associated with the generators chosen to handle the flow limit. The values associated with these generators are set to 1 (generator assigned to increase generation, 'INC' redispatch [12]) and –1 (generator assigned to decrease generation, 'DEC' redispatch [12]), whereas the rest of the vector is zero. Hence, as one generator is assigned to increase generation by TS, another generator is assigned to reduce its generation by the same TS. Notice that, in the examples in this paper, only two generators are chosen for redispatch. However, using a larger number of generators is straightforward. Likewise, the method may handle more than one flow limit. In this case, for each flow limit analyzed, a new row, a new column, and a new scalar variable should be added. Vector $k1$ corresponds to the predetermined generation increase direction vector ΔP_g. $FF1$ and $kF1$ correspond to parameterization equation in a continuation power flow. Vectors $k1$, $FF1$ and the scalar $kF1$ are shown for clarity, but there is never need to solve a set of equations larger than the optimal set. Vectors $k2$ and $FF2$ are added to the set of equation as soon as a flow limit is identified and from that point on remain in the set of equations. The new set of equations obtained is solved by Newton-Raphson method. Its output consists of the regular state variables (phase angles at PV and PQ buses and voltage magnitudes at PQ buses) and the active power deviation at the generators chosen to remove the overload (TS). From the perspective of this paper, a constraint on voltage limits can be handled the same way as a flow limit: by the inclusion of an extra equation for |V| related to the redispatch of the generator pairs.

5.1 The choice of a proper generator pair

The factors influence the limiting values of line flows are [2]: thermal limit (I^2 limit), small-signal (steady state) stability limit (P_{flow} limit), voltage difference limit (S_{flow}

limit). To be able to handle these limits, we define four options for active power generation redispatch:

- "Operator-specified",
- "Most effective",
- "Sufficient",
- "Cheapest".

"Operator-specified" redispatch. This option completely relies on user (operator) experience. A generator pair is simply chosen by the operator. This enables one to reproduce a real situation, when the operator is assigned to pick a generator pair. This redispatch option can be: "Chunk" and "Continuous". Difference between two "Operator-specified" options is illustrated in Figure 2.

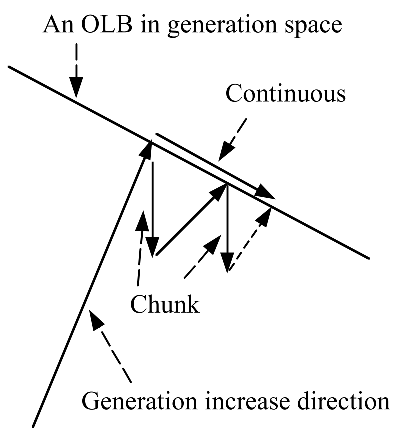

Figure 2 Two options of "Operator-specified" redispatch

In "Chunk" option the operator simply specifies a generator pair and the amount of active power to be redispatched, system is not "moving" along reached OLB (notice in this case is not necessary to include an additional equation to the system equations). In "Continuous" redispatch the operator specifies a generator pair, system is "moving" along reached OLB keeping reached limit at limiting value.

"Most effective" redispatch. This option is eminently technical. The choice of a proper generator pair is based on sensitivities of the line flow in relation to each system generator. Here we define Flow Distribution Factors (FDF) as a static measure of the percent impact of a change in generation on a line flow "with respect to" the power flow model's swing bus. FDF requires determination of a line flow equations Jacobian matrix, that relates the flows at either end of a line to changes in voltage magnitudes and angles. The calculation of the Jacobian matrix is based on a sparse and Matlab vectorized representation of the line flows. The power flow of each line can be calculated by

$$\overline{S}_L = \overline{V}_L \bullet \overline{I}_L^* \qquad (4)$$

where:

$\overline{V}_L$ - vector of line voltages ($\overline{V}_L = A^T \overline{V}$),

$\overline{I}_L$ - vector of injected current of line ($\overline{I}_L = \overline{Y}\overline{V}_L$),

$A_{N\times 2L}$ - an associate relationship matrix,

$Y_{2L\times 2L}$ - the primary admittance matrix in which the diagonal elements are small admittance matrix (2-port representation of branch and transformer),

$\bullet$ - is defined as the point wise multiplication of two vectors.

N denotes the number of buses, and L denotes the number of 2 terminal lines in the system. Equation (4) also can be expressed as

$$\overline{S}_L = diag(\overline{V}_L)\overline{I}_L^* = diag(\overline{I}_L^*)\overline{V}_L \qquad (5)$$

The first partial derivatives of line flow in relation to V and δ can be obtained by

$$\frac{\partial \overline{S}_L}{\partial V} = diag(\overline{V}_L)\frac{\partial \overline{I}_L^*}{\partial V} + diag(\overline{I}_L^*)\frac{\partial \overline{V}_L}{\partial V} \qquad (6)$$

$$\frac{\partial \overline{S}_L}{\partial \delta} = diag(\overline{V}_L)\frac{\partial \overline{I}_L^*}{\partial \delta} + diag(\overline{I}_L^*)\frac{\partial \overline{V}_L}{\partial \delta} \qquad (7)$$

$$\frac{\partial \overline{I}_L^*}{\partial V} = \overline{Y}^* \frac{\partial \overline{V}_L^*}{\partial V}, \quad \frac{\partial \overline{I}_L^*}{\partial \delta} = \overline{Y}^* \frac{\partial \overline{V}_L^*}{\partial \delta} \qquad (8)$$

$$\frac{\partial \overline{V}_L}{\partial V} = A^T \frac{\partial \overline{V}}{\partial V} = A^T diag(e^{j\delta}) \qquad (9)$$

$$\frac{\partial \overline{V}_L}{\partial \delta} = A^T \frac{\partial \overline{V}}{\partial \delta} = A^T diag(jV) \qquad (10)$$

where: $e^{j\delta} = \frac{\overline{V}}{V}$ pointwise.

For known power flow Jacobian matrix and line flow Jacobian matrix FDF can be calculated by

$$FDF = (J_{flow} e_1)(J^{-1} diag(e_2)) \qquad (11)$$

where J_{flow} is either the from or to end line flow generic Jacobian matrix, J is power flow Jacobian, e_1 is the vector with all entries equal to zero except the entry corresponding to the limited line equal to 1, elements of e_2 are equal to 1 for all non-slack PV buses and the rest of elements equal to zero. This calculation allows determination of FDF for all mentioned flow limits types. All matrices involved are sparse so that sparse techniques can be applied in the calculations. The outcome of FDF determination is the list of generators most effective to handle the line flow limit.

"Sufficient" and "Cheapest" redispatch. After the sensitivity approach described above is executed, a proper generator pair for "sufficient" and "cheapest" redispatch is chosen according to the formulation:

$"Sufficient" = "Most-effective" \times (P_{\max} - P_{actual})$; for 'INC',

$"Sufficient" = "Most-effective" \times (P_{actual} - P_{\min})$; for ‘DEC’,
$"Cheapest" = "Sufficient" \times \$$; for both ‘INC’ and ‘DEC’.

6 Test results

The proposed methodology is demonstrated on the IEEE-39 and IEEE-118 bus systems and the results of five tests are presented. In the case when the flow limits in all transmission lines are taken to infinity (no flow limits), the system reaches voltage collapse limit, and the maximum loadabilities are 11998.54 MW (IEEE-39) and 19250.38 MW (IEEE-118). Two tests were carried out on the IEEE-39 bus system. In the first case the flow limit in line 9 (between buses 4 and 14) is reached at load level 7029.87 MW. “Operator-specified” (continuous) redispatch option is used to handle the reached flow limit. The chosen generator pair is (‘INC’=35) and (‘DEC’=32). Figure 3 illustrates the obtained results.

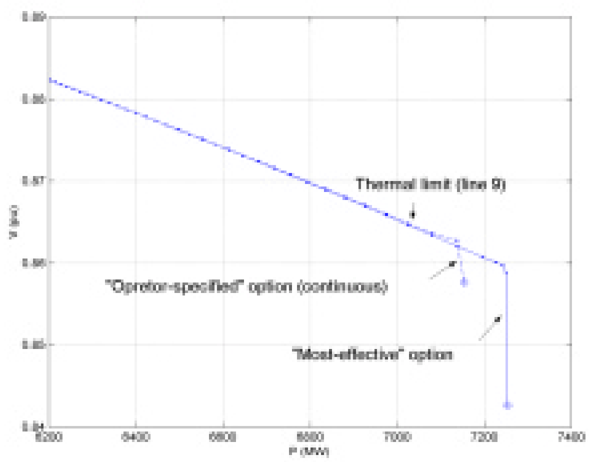

Figure 3 PV curve of Bus 26 (IEEE-39 bus system)

In the second case, with the same line flow limit reached at the same load level, “Most effective” redispatch option is used, and results are also presented in Figure 3.
The proper generator pair is ‘INC’=30, ‘DEC’=32. Figure 3 shows that in the case of “Most effective” redispatch option the system can be steered further than in case of “Operator-specified” redispatch, because this option is eminently technical. In both cases the system meets the voltage collapse limit at the end. “Sufficient” redispatch option is used in the test carried out on IEEE-118 bus system. Figure 4 shows the PV curve associated with the system critical bus at the bifurcation point (bus 95).

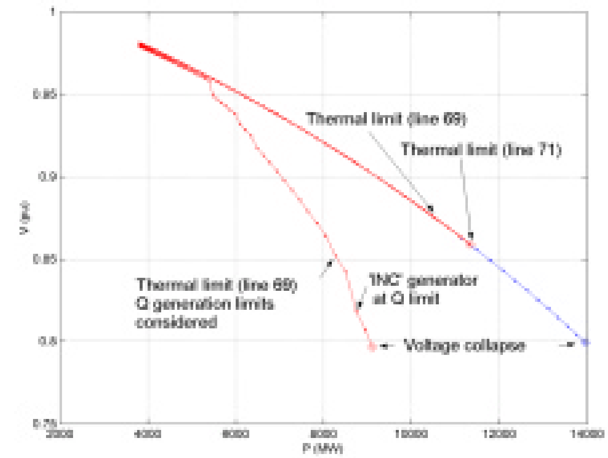

Figure 4 PV curve of Bus 95 for various system conditions

The flow limit in line 69 (between buses 48 and 49) is reached at the load level 10488.35 MW. The generator pair to act according to “Sufficient” redispatch is ‘INC’=46, ‘DEC’=49. At load level 13997.63 MW the system reaches the VC limit. Two additional tests were carried out on the same test system, one by using “Cheapest” redispatch option (linear generation costs are considered) with the same assumptions (P and Q generation limits are not considered), and another one with Q generation limits consideration and by using “Sufficient” redispatch option. The results are presented in Figure 4. For this particular case, the same generator pair has been chosen to handle the reached flow limits. To make a difference from the previous test, the second flow limit (line 71) is reached at load level 11334.21 MW. If Q generation limits are considered, the flow limit in the same line is reached at load level 8237.73 MW. The same generator pair has been chosen again (“Sufficient” redispatch option). At load level 8726.30 MW, ‘INC’ generator reaches its upper Q limit, which results in a rapid voltage drop. The next generator picked to increase output is generator 69, and eventually the system reaches VC limit at load level of 9115.18 MW.

One important aspect of this problem must be considered: the “lack” of generators available to handle the flow limits. This happens because generators electrically close to the limited line have already reached their P limits. The FDF calculation indicates still available generators most likely to act, but the “recommended” generation pair is electrically far away from the flow limited line. To demonstrate the capabilities of the enhanced point of collapse power flow, another test was carried out with the IEEE 118 bus system with all limits considered.

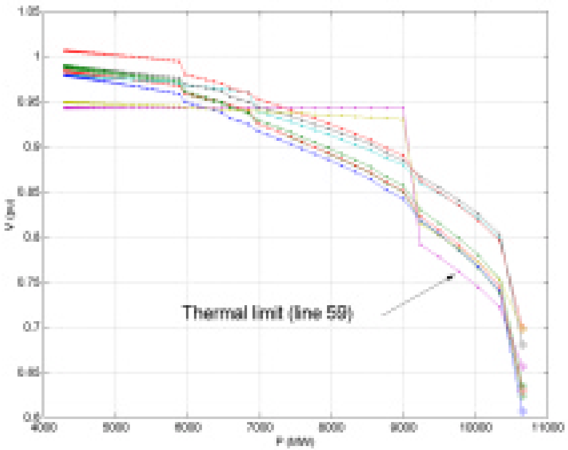

Figure 5 PV curves of some system buses (all limits considered)

At the load level amount of 9739.98 MW the thermal limit in transmission line 43-44 is identified. The redispatch option employed is "Operator-specified". User choice of a proper generator pair is based on the inspection of the system topology, and generator 54 is chosen as the 'INC', whereas generator 40 is assigned to 'DEC'. This is because these generators are the closest ones to the ending buses of the limited line. The system reaches a voltage stability limit at the load level of 10156.92 MW. In the all tests it has been assumed that load increase direction is pre-specified. In the real world loads vary in a not fully predictable manner. Load variations tend to be correlated to time of day and to weather (particularly temperature). Significant system demand variations tend to be "slow" in time frame of minutes and hours. The methodology is capable of handling changes in load direction and with a reasonable load forecast model incorporated, and with already incorporated sparse matrix and vectorized computing methods, on-line implementation of the methodology will be possible.

7 Relationship to Market Redispatch

According to the NERC document [12] that contains the specific procedure outlining an improved market redispatch pilot program in the Eastern Interconnection for the period June 1, 2000, though December 31, 2000, the procedure for market redispatch (MRD) consists of six steps:

1. Posting of advance information by Security Coordinators (SC) or Transmission Providers (TP), about active limiting flowgates (FW).
2. Determination of the generators to be redispatched by Purchasing and Selling Entities (PSE).
3. Transmittal of the market redispatch tag by PSE.
4. MRD transaction impact analysis by the SC and TP.
5. Process for initiating a MRD transaction.
6. Monitoring and reloading.

The main purpose of market redispatch is to protect the original impacting transaction, by creating in advance a bilateral redispatch transaction (by one or more PCE) that create a counter-flow over a potentially constrained flow gate (FG), from curtailment which would impose constraint on a FG. The enhanced point of collapse power flow, used in this paper, provides enough information for solving problems in steps 1 and 2. Rapid and efficient computation of PTDF, OTDF, FDF and TS (in document [12] termed Generation Shift Factors, GSF), in the proposed methodology, is in the heart of the steps 1 and 2, and the methodology can be used by SC or TP as a useful helper for operating a power system within its security constraints. Creating a counter-flow over potentially constrained FG does not mean the keeping a line flow exactly on the limit, but the proposed methodology is capable of keeping line flows at any pre-specified value.

8 Conclusions

Explicit specification of generation redispatch strategies possible for flow limit enforcement has been presented in this paper. The sparse, vectorized Newton implementation used in the point of collapse power flow has been easily extended by explicit consideration of a flow limit equation in the set of system model equations. This new feature renders the tool as a powerful and accurate helper for operating a power system within its security constraints. For the line flow limits problem the operator is allowed to identify the most effective generator pair according to four different options, based on topology analysis, sensitivity studies, generator margins, or cost considerations. Only thermal line flow limit has been considered in the paper and including other two limits is straightforward. The presented features provide important new insights in the area focused in the paper. The results carried out with the help of the IEEE-39 and IEEE-118 bus system indicate that the methodology is effective.

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Appendix A: Derivation of $FF2$, $k2$ and FDF

In section 5 the generic line flow Jacobian matrix J_{flow} has been introduced. The matrix has the form

$$J_{flow} = \left[\frac{\partial l_L}{\partial V}, \frac{\partial l_L}{\partial \delta} \right] \qquad \text{(A1)}$$

where l denotes generic line flow equation, and L denotes the number of 2 terminal lines in the system. Depending of the limit considered, l becomes I^2, P_{flow}, or S_{flow}. The elements of the matrix are calculated as

$$\frac{\partial I_L^2}{\partial V} = \frac{\partial I_L}{\partial V} I^* + I \frac{\partial I_L^*}{\partial V} = \overline{Y} \left[\frac{\partial \overline{V}_L}{\partial V} \overline{Y}^* \overline{V}_L^* + \overline{V}_L \overline{Y}^* \frac{\partial \overline{V}_L^*}{\partial V} \right], \text{(A2)}$$

$$\frac{\partial I_L^2}{\partial \delta} = \frac{\partial I_L}{\partial \delta} I^* + I \frac{\partial I_L^*}{\partial \delta} = \overline{Y} \left[\frac{\partial \overline{V}_L}{\partial \delta} \overline{Y}^* \overline{V}_L^* + \overline{V}_L \overline{Y}^* \frac{\partial \overline{V}_L^*}{\partial \delta} \right], \text{(A3)}$$

$$\frac{\partial P_L}{\partial V} = real(\frac{\partial \overline{S}_L}{\partial V}), \quad \frac{\partial P_L}{\partial \delta} = real(\frac{\partial \overline{S}_L}{\partial \delta}), \qquad \text{(A4)}$$

$$\frac{\partial S_L}{\partial V} = Y^2 \frac{\partial \overline{V}_L}{\partial V}, \quad \frac{\partial S_L}{\partial \delta} = Y^2 \frac{\partial \overline{V}_L}{\partial \delta}. \qquad \text{(A5)}$$

The expressions for $\frac{\partial \overline{V}_L}{\partial V}$ and $\frac{\partial \overline{V}_L}{\partial \delta}$ are given in section 5. Four nonzero elements of $FF2$ can be obtained by

$$FF2 = J_{flow} e_{11} \qquad \text{(A6)}$$

where e_{11} is the vector with all entries equal to zero except that four entries corresponding to ending nodes of limited line are equal to 1.

Let M generators (according to the FDF calculated by equation 11 with the details given here) be assigned to participate in flow limit handling. Vector k2 contains M nonzero elements. The nonzero value corresponding to $i-th$ 'INC' generator is calculated by

$$k2_i = \frac{FDF_i}{\sum_{j \in 'INC'} FDF_j}, \qquad \text{(A7)}$$

and, for corresponding 'DEC' generator,

$$k2_n = \frac{FDF_n}{\sum_{j \in 'DEC'} FDF_j}. \qquad \text{(A8)}$$

Appendix B: Relation to work in references [8] and [13]

The paper in [13] dealt with the problem of generic transfer margin computation and sensitivity of transfer capability margins by using fast sensitivity formula. The methodology presented in that paper is capable of finding an equilibrium solution in the limited case (limits on line flows, voltage magnitudes, generator VAR outputs and voltage collapse) by adding to the model an applicable equation for the binding limit. The methodology presented in this paper is based on the same principle. Namely, inclusion of flow limit equations in point of collapse power flow is nothing else than adding an applicable equation for the binding limit. In addition, the methodology is capable to find a sequence of equilibrium solutions in the limited case. In this paper voltage collapse is considered as a hard system limit, but in practice it is often not the case. Considerable work has been done in determination of the control actions to alleviate or avoid the problem, as in [8]. Computing the control direction in control space (active and reactive powers of loads and generators, settings of tap changing transformers, and shunt capacitor devices) to avoid saddle node bifurcation and voltage collapse, relies on iterating a continuation power flow to find the closest bifurcation point and the vector normal to the bifurcation hypersurface. The methodology presented in this paper is also capable of handling VC limits by active generation redispatch. We restrict a control space to active powers of generators (in fact only two generators throughout this paper) and calculate the normal vector at a bifurcation point. Notice, in this case it is not necessary to add a new equation to the system model (the aim is not to keep the system at bifurcation point), at least not in the same way presented in the paper because vectors $k1$ and $FF1$ take care of this.

Normal vector entries give the information about the generators to be used in redispatching. To demonstrate this capability another test was carried out with help of IEEE-118 bus system. The system reaches VC at load level of 10386.5 MW. According to the normal vector calculation [8], a generator pair to act is 'INC'=54, 'DEC'=40. Redispatch ("Operator-specified"- "Chunk") is successful up to load level of 10720.34 MW where the system reaches the ultimate VC.

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Sensitivity of transfer capability margins with a fast formula

Scott Greene, Ian Dobson, and Fernando L. Alvarado

Abstract— **Bulk power transfers in electric power systems are limited by transmission network security. Transfer capability measures the maximum power transfer permissible under certain assumptions. Once a transfer capability has been computed for one set of assumptions, it is useful to quickly estimate the effect on the transfer capability of modifying those assumptions. This paper presents a computationally efficient formula for the first order sensitivity of the transfer capability with respect to the variation of any parameters. The sensitivity formula is very fast to evaluate. The approach is consistent with the current industrial practice of using DC load flow models and significantly generalizes that practice to more detailed AC power system models that include voltage and VAR limits. The computation is illustrated and tested on a 3357 bus power system.**

Keywords— **sensitivity, power system security, power system control, power transmission planning, optimization**

I. INTRODUCTION

Transfer capability indicates how much a particular bulk power transfer can be changed without compromising system security under a specific set of assumptions. The increased attention to the economic value of transfers motivates more accurate and defensible transfer capability computations.

A variety of applications in both planning and operations require the repetitive computation of transfer capabilities. Transfer capabilities must be quickly computed for various assumptions representing possible future system conditions and then recomputed as system conditions change. The usefulness of each computed transfer capability is enhanced if the sensitivity of the transfer capability is also computed [15], [10]. This paper shows how to quickly compute these sensitivities in a general and efficient way. The sensitivities can be used to estimate the effect on the transfer capability of variation in simultaneous transfers, assumed data, and system controls. A web site [6] is available to calculate these sensitivities on sample power systems and further illustrate their use.

While there is general agreement on the overall purpose and outline of transfer capability determination, the precise requirements for such computations vary by region and are evolving. In this paper we focus on the fast computation of the sensitivity of the transfer margin, not the computation of the transfer margin itself. However, to explain the sensitivity computation we need to first discuss a generic transfer margin computation. The sensitivity computation is largely independent of the method used to obtain the transfer margin.

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II. A GENERIC TRANSFER MARGIN COMPUTATION

We assume that an initial transfer margin computation has established:

1. A secure, solved base case consistent with the study operating horizon.
2. Specification of transfer direction including source, sink, and loss assumptions.
3. A solved transfer-limited case and a binding security limit. The binding security limit can be a limit on line flow, voltage magnitude, voltage collapse or other operating constraint. Further transfer in the specified direction would cause the violation of the binding limit and compromise system security.
4. The transfer margin is the difference between the transfer at the base case and the limiting case.

Calculations of Available Transfer Capability (ATC), Capacity Benefit Margin (CBM), and Transfer Reliability Margin (TRM) typically require that this generic transfer margin computation be repeated for multiple combinations of transfer directions, base case conditions, and contingencies [13], [15].

The generic transfer margin computation can be implemented with a range of power system models and computational techniques. One convenient and standard practice is to use a DC power flow model to establish transfer capability limited by line flow limits. The limiting cases are then checked with further AC load flow analysis to detect possibly more limiting voltage constraints.

Alternatively, a detailed AC power system model can be used throughout and the transfer margin determined by successive AC load flow calculations [10] or continuation methods [2], [3], [1], [16]. A related approach [e.g., EPRI's TRACE] uses an optimal power flow where the optimization adjusts controls such as tap and switching variables to maximize the specified transfer subject to the power flow equilibrium and limit constraints. The formulations in [10] and [18] show the close connection between optimization and continuation or successive load flow computation for transfer capability determination. The sensitivity methods of this paper are applicable to transfer margins computed by optimization, continuation or other methods. The implementation of the sensitivity formula can take advantage of numerical by-products of common sequential linear programming techniques.

Methods based on AC power system models are slower than methods using DC load flow models but do allow for consideration of additional system limits and more accurate accounting of the operation guides and control actions that accompany the increasing transfers. Under highly stressed conditions the effects of tap changing, capacitor switching, and generator reactive power limits become significant. A combination of DC and AC methods may be needed to achieve the correct tradeoff between speed and accuracy. The methods in this paper account directly for any limits which can be deduced from equilibrium equations such as DC or AC load flow equations or enhanced AC equilibrium models.

III. SENSITIVITY COMPUTATION

A. System Modeling

Assume a general power system equilibrium model written as n equations:

$$0 = f(x, \lambda, p)$$

where
x is an n dimensional state vector that includes voltage magnitudes, angles, branch flows, and generator MW and MVAR outputs.
λ is a vector of generator MW output set points and/or scheduled net area exports.
p is a parameter vector including regulated voltage set points, generator load sharing factors, load and load model parameters and tap settings.
The limits on line flows, voltage magnitudes, or generator VAR outputs are modeled by inequalities in the states:

$$x_i^{\min} \le x_i \le x_i^{\max} \qquad , i = 1, \ldots, n.$$

Due to the modeling of operator actions and generator limits, the equilibrium equations and the physical quantities represented by the x and p vectors can change under varying conditions and transfer levels. For example, when a non-slack generator is operated within its reactive power limits, the reactive power output and angle at the generator bus are components of x and the regulated bus voltage and real power output are components of p. However, when the same generator is at a reactive power limit, the generator bus voltage and angle are components of x and the real and reactive power output are components of p.
Base case: The base case specifies the nominal value λ_0 of the generator outputs and net area exports.
Transfer specification: The transfer is specified by changes to the vector λ. The transfer direction describes how λ changes as the transfer increases so that

$$\lambda = \lambda_0 + k\,t$$

where t is the transfer amount and k is a unit vector describing the transfer direction. For the simple case of net exports increasing from one area matched by reduction in net export from another area, the transfer direction k is a column vector with 1 in the row corresponding to the source area export equation and -1 in the row corresponding to the sink area export equation. For transfers specified by changes in individual bus injections, k is a column vector with positive entries at the source buses and negative entries at the sink buses.
Transfer-limited case: Identification of a solved transfer-limited case yields an equilibrium solution (x_*, λ_*, p_*) and an additional constraint referred to as the *binding limit*. The equilibrium equations that model the power system *at the binding limit* are written

$$0 = F(x, \lambda, p) \tag{1}$$

When a limit is encountered, one of the limit equations $x_i = x_i^{\min}$ or $x_i = x_i^{\max}$ holds for some i. We write the applicable equation for the binding limit in the general form

$$0 = E(x, \lambda, p) \tag{2}$$

The form (2) also encompasses more general limits. At the binding limit

$$\begin{aligned} F(x_*, \lambda_*, p_*) &= 0 \\ E(x_*, \lambda_*, p_*) &= 0 \end{aligned}$$

Transfer margin: The transfer margin is the change in the transfer between the base case and the transfer-limited case. Since $\lambda_* = \lambda_0 + kT$, the transfer margin is T.

B. Sensitivity Formula

Once the binding limit and the corresponding transfer-limited solved case have been found, the sensitivity of the transfer margin T can be evaluated. The sensitivity of T to the parameter p, often written as $\frac{\partial T}{\partial p}$ and here written as T_p, is computed using a formula derived in Appendices -A and -B:

$$T_p = \frac{-w \begin{pmatrix} F_p \\ E_p \end{pmatrix}\Big|_{(x_*,\lambda_*,p_*)}}{w \begin{pmatrix} F_\lambda k \\ E_\lambda k \end{pmatrix}\Big|_{(x_*,\lambda_*,p_*)}} \tag{3}$$

where
• F_p and E_p are the derivatives of the equilibrium and limit equations with respect to p.
• $F_\lambda k$ and $E_\lambda k$ are the derivatives of the equilibrium and limit equations with respect to the amount of transfer t.
• w is a nonzero row vector orthogonal to the range of the Jacobian matrix J of the equilibrium and limit equations, where

$$J = \begin{pmatrix} F_x \\ E_x \end{pmatrix}\Big|_{(x_*,\lambda_*,p_*)}$$

The row vector w is found by solving the linear system

$$wJ = 0 \tag{4}$$

Since J has one more row than column, there is always a nonzero vector w that solves (4). J generically has full column rank, so that w is unique up to a scalar multiple. The sensitivity T_p computed from (3) is independent of the scalar multiple.

The first order estimate of the change in transfer margin corresponding to the change in p of Δp is

$$\Delta T = T_p \, \Delta p \qquad (5)$$

If the binding limit is an immediate voltage collapse due to a reactive power limit [5], then the analysis of this paper applies with the limit equation (2) becoming $Q_i = Q_i^{\max}$. If the binding limit is voltage collapse due to a fold bifurcation, the sensitivity formula of [8] applies.

C. Computational Efficiency

Once the transfer-limited solution is obtained, the margin estimates corresponding to varying a large number of different parameters can be obtained for little more computational effort than solving the sparse linear equations (4) for w. Solving (4) is roughly equivalent to one Newton iteration of a load flow solution. Note that w need only be computed once but can be used to find the sensitivity with respect to any number of parameters. If a sequential LP is used to determine the transfer margin as part of an optimization program, then w is found from the Lagrange multipliers obtained at the last LP solution. The remaining computations (3) and (5) needed for the estimates require only sparse matrix-vector multiplications.

The Jacobian matrix J in (4) is available, often in factored form, from the computation of the transfer-limited solution by Newton based methods. The matrix F_p in (3) is different for each parameter p but its construction is a simple sparse index operation, especially when the parameters appear linearly.

The sensitivity of the transfer capability with respect to thousands of changes in load, generation, interarea transfers, or voltage set points can be obtained in less time than a single AC load flow solution.

IV. 3357 BUS EXAMPLE

The application of sensitivity formula (3) is illustrated using a 3357 bus model of a portion of the North American eastern interconnect. The model contains a detailed representation of the network operated by the New York independent system operator and an equivalent representation of more distant portions of the network. From a base case representative of a severely stressed power system, small increases in transfer between Ontario Hydro and New York City lead to low voltages, cascading generator reactive power limits, and finally voltage instability. The sensitivity formulas are used to identify effective control action to avoid low voltage and VAR limit conditions, and to estimate the effects of variation in transfers and loading on the security of the system.

Base case: The base case is motivated by a scenario identified as problematic in the New York Power Pool summer 1999 operating study. The loss of two 345 KV lines, Kintigh-Rochester and Rochester-Pannell Road during high west to east transfer leads to low voltage conditions at the Rochester 345 kV bus. At the base case solution, the voltage at the Rochester 345 kV bus is 333 kV, slightly above the 328 kV low voltage rating.

Limiting events: From the base case, a sequence of AC load flow solutions are obtained for increasing levels of export from Ontario Hydro and increasing demand in the New York City zone. A 100 MW increase in this transfer results in the voltage at the 345 kV Rochester bus reaching its low voltage rating of 328 kV. Additional transfer leads to several low voltages and nine additional generating units reaching maximum VAR limits. Finally, for transfer of 140 MW beyond that corresponding to the Rochester voltage limit, a reactive power limit at one of the Danksammer generating units leads to immediate voltage instability [5]. (System behavior under the stressed conditions is unstable without voltage regulation at Danksammer.)

TABLE I

Net zone exports in MW at base case, the initial voltage limit at the Rochester 345KV bus, and the final reactive power limit at Danksammer.

ZONE	net export base case	net export voltage limit	net export VAR limit
NYC	–4806	–4906	–5046
OH	4080	4180	4320
HQ	976	976	976
PJM	–3422	–3422	–3422
ISO-NE	–28	–28	–28

Table I shows the net exports for five of the zones at the base case and at two different limits. The transfer margin to the voltage limit is 100 MW and the transfer margin to the critical VAR limit is 240 MW. Since it is of interest how avoiding the low voltage limit also improves the margin to voltage instability, we compute the sensitivities of both these margins.

A. Sensitivity to regulated voltage set points

The sensitivity of the transfer margins to the Rochester voltage limit and the Danksammer VAR limit with respect to all parameters is obtained using formula (3). Ranking of all the NY ISO generator buses according to the sensitivity of the transfer margins with respect to regulated generator voltages indicates that the regulated voltage with the greatest effect on the transfer margin to the Rochester voltage limit and the second greatest effect on the margin to the Danksammer VAR limit is the Hydro facility in Niagara.

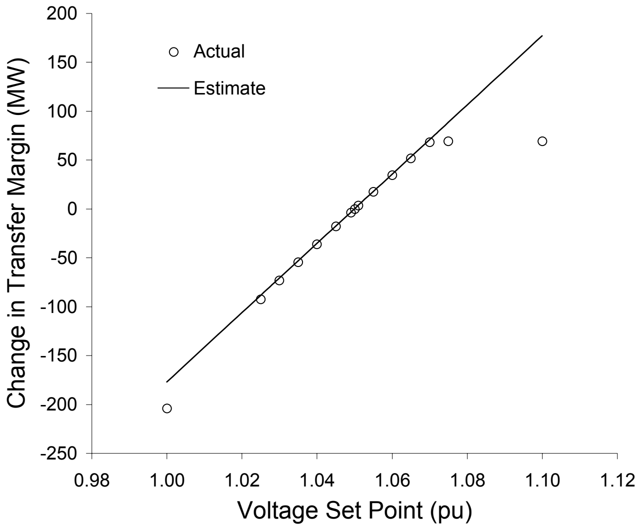

Fig. 1. Effect of regulated output voltage on margin to voltage limit.

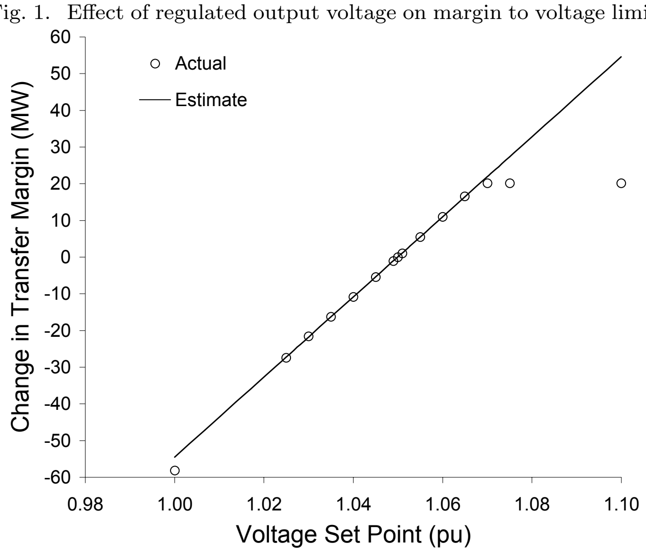

Fig. 2. Effect of regulated output voltage on margin to critical VAR limit.

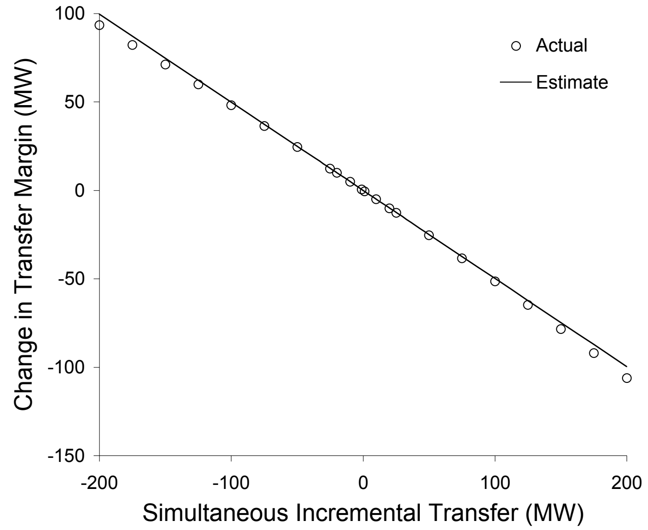

Fig. 3. Effect of simultaneous transfer on margin to voltage limit.

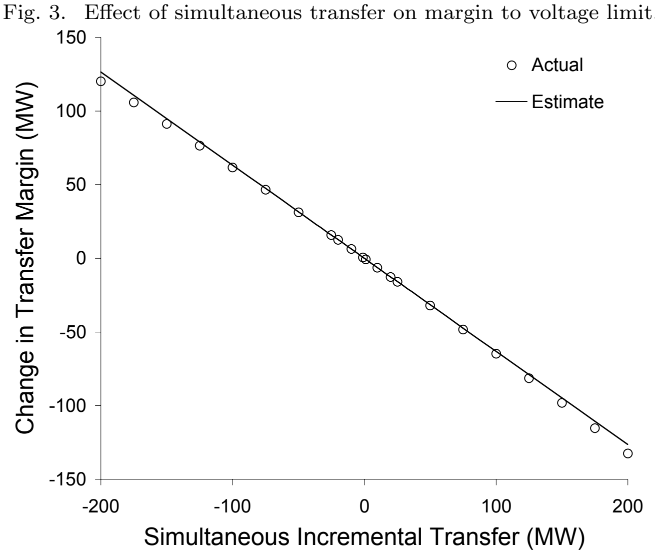

Fig. 4. Effect of simultaneous transfer on margin to critical VAR limit.

Fig. 1 shows the linear estimate for the change in transfer margin to the voltage limit as a function of the voltage set point at the Niagara generator. The estimates are compared with actual values computed by AC loadflow analysis represented by the circles in Fig. 1. The actual values are obtained by incrementing the voltage set point and rerunning the transfer capability calculation. In effect, the incremental variation method of [9] is used to check the sensitivity formula. Fig. 2 compares the linear estimate with actual values computed by AC loadflow analysis for the change in the transfer margin to the Danksammer VAR limit as a function of the Niagara voltage set point. Figs. 1 and 2 show that the estimates are accurate for a ±5% variation in the regulated output voltage of the Niagara unit. Note that for both limits, setting the voltage set point greater than 1.07 pu does not improve the margin as predicted because at that voltage the generator reactive power output reaches its maximum before the transfer limit is encountered.

B. Sensitivity to Simultaneous Transfers

One concern is the effect of simultaneous transfers on the computed transfer margins. Figs. 3 and 4 show the effects on the voltage and VAR limited transfer margins of a simultaneous Hydro Quebec to PJM transfer. The simultaneous transfer affects the VAR limit more than the voltage limit, and the sensitivity based estimates are accurate for a ±200 MW transfer variation, which is a 20% variation in export from Hydro Quebec.

C. Sensitivity to Load Variation

Another concern is load forecast error. For example, consider the effect of load variation in the Albany region on the transfer margins. The real and reactive power loads in Albany are changed keeping constant power factor. The estimates are compared with the actual values computed directly from AC loadflow analysis in Figs. 5 and 6. The results are very accurate for ±200 MW total load variation, but less accurate for ±400 MW. The base case Albany zone

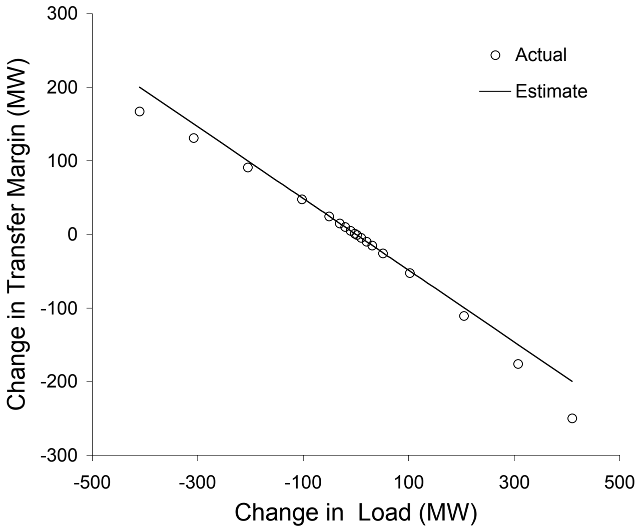

Fig. 5. Effect of Albany loading on margin to voltage limit.

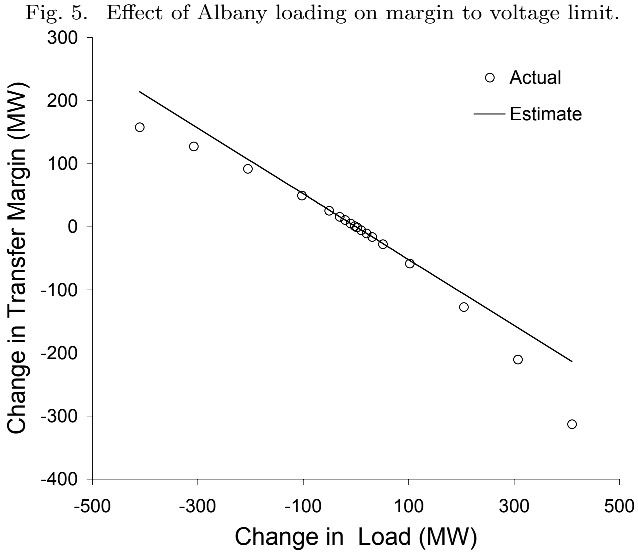

Fig. 6. Effect of Albany loading on margin to critical VAR limit.

load is 2000 MW.

All the results confirm the accuracy of the formula in predicting the transfer margin when small changes are made in a parameter. For some parameters, the transfer margin is accurately predicted for large[1] changes in the parameter. The range of parameter variation for which the prediction of transfer capability is accurate depends on the parameter considered, but generally is sufficiently large to support the usefulness of the first order approximation. Two possible sources of error in predicting the transfer margin for large parameter changes are:

- Nonlinearity. For a fixed power system equations, the transfer margin varies nonlinearly with the parameter. For example, this is evident in the curvature of the actual results in Figs. 5 and 6.
- Structural changes. As the parameter changes from its nominal value, the power system equations change when variables reach limits. After the equations change, the estimated changes in transfer margin computed with the equations valid at the nominal parameter value can be inaccurate. For example, this is evident in the sudden change in the actual results in Figs. 1 and 2 when a generator reactive power limit is encountered. It is clear that proximity of the transfer limited case to limits can in some cases limit the accuracy of the estimated changes in transfer margin. This proximity can be detected by the additional computation of state variable sensitivities suggested in Appendix -A.

[1]We clarify meanings of "small" and "large": From a mathematical perspective,"small" means "infinitesimally small". From an engineering perspective, "small" can, for example, be 1 MW for power variations and 0.1% for changes in voltage magnitude. Thus "small" corresponds to parameter changes for which the first order linearization will produce very accurate results. "Large" means not small.

V. Handling Multiple Limits

A simple approach computes the sensitivity of the transfer margin to the single binding limit. In practice, particularly when the power system is uniformly and highly stressed, there are often other limits encountered just after the binding limit.

For example, Fig. 7A illustrates the next limit encountered at N if the binding limit at M is neglected. This next limit can be computed by running the continuation past the binding limit. Fig. 7A shows that if the parameter is increased past 0.56, the next limit becomes the binding limit. In the situation of Fig. 7A, the sensitivity of the transfer margin to both the binding limit and the next limit can be computed using the methods of this paper and the resulting linear estimates of the changes in these margins are illustrated in Fig. 7B. For power system examples of this computation see [6].

Thus in the presence of multiple limits close to the binding limit, we recommend that the sensitivity of the corresponding transfer margins also be computed. Then the power system can be steered away from several security limits that may become binding. Finding the transfer margin sensitivity at each further limit requires re-computation of the transfer limited case. This is usually much quicker than the original computation of the binding transfer limited case, because if the further limit is relevant, it must occur soon after the binding limit. However, the re-computation of each transfer limited case is significantly more expensive than the sensitivity computation for each limit. Prediction of which voltage magnitude and line limits will occur soon after a binding limit can be done using the additional computation of state variable sensitivities suggested in Appendix -A.

VI. Related Work

The primary tool used in industry for computing transfer capability margins is the DC loadflow model with PTDF and OTDF computations (e.g., PTI program MUST [12]). It can be shown [7] that the sensitivity formula (3) reduces to PTDFs and OTDFs for the appropriate DC load flow models and this is illustrated in Appendix -C. Thus this paper significantly generalizes standard sensitivity methods to encompass more accurate transfer capability calculations on more detailed models. In particular, account can be taken of power system nonlinearity, operator and auto-

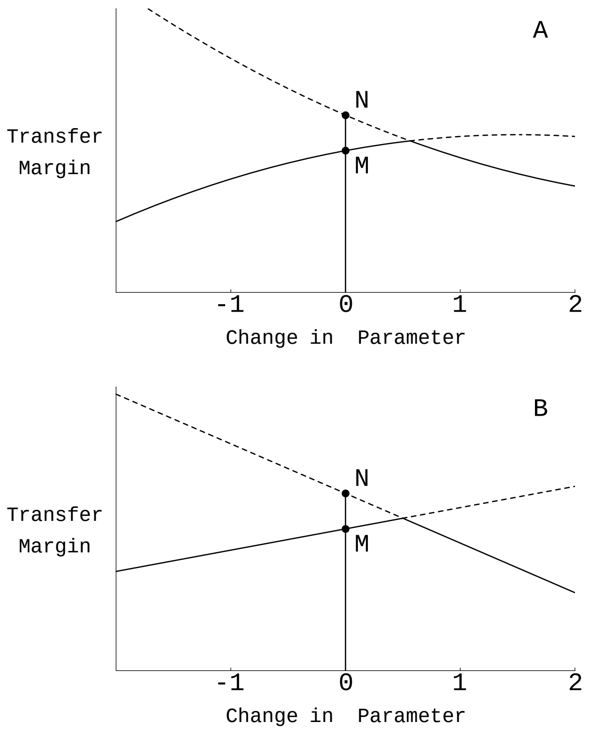

Fig. 7. Effect of parameter change on the next limit (dashed line) encountered just after the binding limit (solid line). A: actual limits; B: estimated limits.

matic control actions, and voltage and VAR limits. The detailed models also expand the range of parameters with respect to which transfer capabilities can be computed. For example, the sensitivity of a line flow limit to reactive power injection can be computed.

There is a close connection between continuation and optimization formulations for computing transfer capabilities. For example, continuation can be viewed as an interior point method of optimizing the amount of transfer. While there can be differences in the assumptions and accuracies of the various continuation and optimization methods of computing the transfer capability, the transfer capability sensitivity formula (3) is unaffected by these choices. The sensitivity formula (3) is derived using both continuation and optimization frameworks in Appendices -A and -B respectively.

Gravener and Nwankpa [10] have also nicely demonstrated the use of transfer margin sensitivities; the difference with this paper lies in the way the sensitivities are computed. In [10], the sensitivities are computed numerically by incrementing the parameter and rerunning whereas we suggest a fast analytical formula for the sensitivities.

The overall margin sensitivity approach which is generalized in this paper arose in the special case and restricted context of loading margins to voltage collapse caused by fold bifurcation [4], [8]. This paper considers transfer margins to general limits other than voltage collapse. The sensitivity of transfer margins to voltage collapse can be easily adapted from [8] and this material special to voltage collapse is not repeated here. The sensitivity formula of [8] differs from formula (3) in that w stands for a different vector and that no event equation is used.[2] [11] demonstrates the use of the margin sensitivity methods of [8] for fast contingency screening for voltage collapse limits only. Testing of fast contingency screening using the more general security limits of this paper is future work.

The transfer capability sensitivity formula (3) was first stated in the workshop [9] and then in the PhD thesis [7]. This paper greatly extends the initial concepts in [9] by deriving the formula, testing it on a realistic power system, and assessing its practicality.

VII. Conclusions

We show how the sensitivity of the transfer capability can be computed very quickly by evaluating an analytic formula at the binding limit. The sensitivities can be used to estimate the effect on the transfer capability of variations in parameters such as those describing other transfers, operating conditions or assumed data. The approach is consistent with current industrial practice using DC load models and significantly generalizes this practice to include more elaborate AC power system models and voltage and VAR limits on power system operation. Once the transfer capability and corresponding binding limit and solved case have been computed, the first order sensitivity of this transfer capability to a wide range of parameters can be quickly computed. These first order sensitivities can contribute to the quick update of transfer capabilities when operating conditions or other transfers change. Moreover, the sensitivities can be used to select operator actions to increase transfer capability.

We conclude that after each computation of a transfer capability, it is so quick and easy to compute sensitivities of that transfer capability that this should be done routinely to extract the maximum amount of engineering value from each computation. In the case of predicting the effects of large parameter changes on transfer margins, even if more than first order accuracy is ultimately required, it is still desirable to first estimate the effects with first order sensitivities.

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Appendices

A. Derivation of sensitivity

Define

$$H(x,\lambda,p) = \begin{pmatrix} F(x,\lambda,p) \\ E(x,\lambda,p) \end{pmatrix}$$

$H(x_*,\lambda_0+kt_*,p)=0$. Assume that H is smooth and assume the generic transversality condition that

$$\begin{pmatrix} H_x & H_\lambda k \end{pmatrix}\big|_* \quad \text{has rank } n+1. \tag{6}$$

Then the implicit function theorem implies that there are smooth functions $X(p)$, $T(p)$ defined near p_* with $X(p_*)=x_*$ and $T(p_*)=t_*$ such that

$$H(X(p),\lambda_0+kT(p),p)=0 \tag{7}$$

Differentiating (7) yields

$$\begin{pmatrix} H_x & H_\lambda k \end{pmatrix}\big|_* \begin{pmatrix} X_p \\ T_p \end{pmatrix} = -H_p|_* \tag{8}$$

There is a nonzero row vector w such that $wH_x|_*=0$. w is unique up to a scalar multiple when $H_x|_*$ has full rank, which is implied by condition (6). Pre-multiplying (8) by w yields

$$wH_\lambda k|_*\,T_p = -wH_p|_* \tag{9}$$

Condition (6) implies that $wH_\lambda k|_*$ is not zero and hence (9) can be solved to obtain (3). The geometric interpretation of the quantities in (3) is that $(wH_\lambda k, -wH_p)$ is the normal vector to the hypersurface in (t,p) space corresponding to the binding limit.

The sensitivity X_p of the states at the binding limit is often useful and this can be obtained by solving (8). For example, $X_p\Delta p$ can be used to screen for cases where new limits would be violated (e.g., $x_i + X_p\Delta p[i] \geq x_i^{\max}$) [7].

B. Derivation of sensitivity in an optimization context

An optimization formulation ([16, chap. 7],[18]) of the transfer margin determination is: Maximize the cost function $T=t$ subject to the equilibrium equations (1) and the limit equations $x_i=x_i^{\min}$ and $x_i=x_i^{\max}$ for all applicable i. This optimization can be solved to find the transfer-limited case equilibrium solution (x_*,λ_*,p_*) and the binding limit (2). In order to use the notation of Appendix -A, note that this solution is also the solution of the optimization: Maximize the cost function $T=t$ subject to

$$H(x,\lambda_0+k\,t,q)=0 \tag{10}$$

$$p-q=0 \tag{11}$$

To be able to quote a common optimization result in the sequel, it is convenient to introduce the parameters p into (10) via the new variables q. The variables are now $X=(x,t,q)^t$. Write

$$L = t - wH - v(p-q) \tag{12}$$

where w and v are row vectors of Lagrange multipliers. Then, at the optimum solution, it is necessary that $0=\frac{\partial L}{\partial X}$, or, equivalently, that

$$wH_x|_*=0 \tag{13}$$

$$1-wH_\lambda|_*k=0 \tag{14}$$

$$wH_p|_*+v=0 \tag{15}$$

Equation (13) is identical to (4), showing that the Lagrange multiplier w must be proportional to the vector w used in the rest of the paper. (The length of the Lagrange multiplier w is fixed by (14).) It is well known in optimization theory (e.g. see [17] or, in the context of applications to minimum cost optimal power flow see [14]) that the sensitivity of the cost function to the constraints is given by the corresponding Lagrange multiplier. Thus $T_p=v$. Applying (15) and then (14) yields

$$T_p = v = -wH_p|_* = \frac{-\,wH_p|_*}{wH_\lambda|_*k} \tag{16}$$

which is identical to the desired formula (3).

C. DC load flow example

We show how the general formula (3) applies in a simple DC load flow example with 6 buses. The slack bus is numbered 0. For the non-slack buses, write $\theta = (\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)^T$ for the angles and $\lambda =$

$(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)^T$ for the power injections. The DC load flow equations are $F(\theta, \lambda) = X\lambda - \theta$. The transfer is from bus 3 to bus 4 so that $k = (0, 0, 1, -1, 0)^T$. The limit on the transfer is overload on line 1–2 so that the limit equation is $E(\theta, \lambda) = b_{12}(\theta_1 - \theta_2) - \lambda_{12\max}$. The parameter is λ_5^0, the base case power injection at bus 5. $F_\theta = -I$ and $E_\theta = (b_{12}, -b_{12}, 0, 0, 0)$ and hence $w = (b_{12}, -b_{12}, 0, 0, 0, 1)$. $F_\lambda = X$ and $E_\lambda = 0$. $F_{\lambda_5^0} = X\,(0, 0, 0, 0, 1)^T$ and $E_{\lambda_5^0} = 0$. The transfer margin T is the increase in transfer from bus 3 to bus 4 which causes the flow limit on line 1–2. Substitution in (3) gives the sensitivity of T with respect to injection at bus 5:

$$T_{\lambda_5^0} = \frac{X_{15} - X_{25}}{X_{13} - X_{23} - X_{14} + X_{24}} = \frac{\rho_{12,5}}{\rho_{12,3} - \rho_{12,4}}$$

where $\rho_{12,m} = b_{12}(X_{1m} - X_{2m})$ is the well known sensitivity of the flow on line 1–2 with respect to power injection at bus m.

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